FORMATION OF SMALL DUST PARTICLES BY BRITTLE DESTRUCTION

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The size distribution of dust particles in nuclear fusion devices is close to the power function. In the paper it is shown that function of this kind can be the result of brittle destruction. From the similarity assumption it follows that the size distribution obeys the power law with the exponent between -4 *and* -1*. The model of destruction has much in common with the fractal theory. The power exponent can be expressed in terms of the fractal dimension. An additional assumption about the structure of fragmentation offers that the exponent is close to* -3*. The exponent for the case of the biggest ball removing equals* -3.4*.*

I. INTRODUCTION

The dust appears in most of nuclear fusion devices due to the plasma-wall interaction. The dust particles pose potential problems to plasma confinement: decrease the plasma temperature, absorb tritium, etc. Therefore, it is necessary to investigate dust formation mechanisms and parameters of the dust. The size distribution is an important characteristic of the dust, as it determines the dust surface area. The latter in turn determines the evaporation rate of the dust and the tritium absorption rate. To our notion, there is no analytical model explaining experimentally observed dust size distributions.

There are two basic distributions commonly used for approximation of experimental results: the log-normal distribution and the power one (the Junge distribution) [1–3]. The Junge distribution was observed in several experiments [2–6] for tungsten and carbon dust in the size range from several nanometers to tens of microns. The exponent of the power distribution was measured to fall between -3.3 and -2.1.

The power size distribution was also observed for the atmospheric dust [1]. The distribution was explained by the model of particle coagulation [7]. Coagulation results in formation of dust agglomerations which are distributed in size according to the power law. In plasma devices, the power size distribution was observed not only for agglomerations, but also for single dust particles in the size range from several nanometers to several microns [2]. Hence it follows that the coagulation model does not explain the dust size distribution in the whole range of sizes.

Four basic mechanisms of dust formation are usually distinguished in plasma devices: flaking of redeposited layers, brittle destruction, condensation from the supersaturated vapor, and growth from hydrocarbon molecules [8]. If the dust grows from the gas (condensation from the supersaturated vapor and growth from hydrocarbon molecules), the size distribution differs from the power law [9]. The typical size of the dust formed by flaking is greater than one micron [10]. At smaller sizes, flaking can be thought of as brittle destruction. In this paper we show that the brittle destruction can indeed provide the power size distribution.

The power law for single dust particles was observed in the wide range between two typical sizes: the grain size of materials and interatomic distance. This fact suggests that the law of material fragmentation is independent of the scale. Moreover, the power distribution itself, which intrinsically has no typical size scale, points to the same conclusion. We find which dust size distributions can be realized under the assumption of scale similarity and show that these are the power ones. The exponent of the distribution function can vary in the range from -4 to -1. With an additional physical assumption, it is possible to show that the exponent is close to -3. We also show the interrelation between the discussed model and fractal theory and connection of the distribution exponent with the fractal dimension.

II. MATHEMATICAL MODEL

First we formulate the mathematical problem. The dust particles produced due to the brittle destruction are fragments of a solid body. The body is to be divided into pieces according to some law. Under the assumption of scale similarity, the division law must be independent of the fragment size. Let us mentally remove the pieces from the body one by one in the order of decreasing size and number them sequentially by the index n. The size distribution function does not depend on the shape of the initial body only for fragments of the size much smaller than the typical size of the initial body. Hence we assume $n \gg 1$.

Denote the volume of the body remained after removal of the n-th piece by V_n, and its surface area by S_n. We can write recurrent expressions relating V_n, S_n, and the characteristic size r_n of the n-th fragment:

$$V_n = V_{n-1} - c_1 r_n^3, S_n = S_{n-1} + c_2 r_n^2, \tag{1}$$

where c_1 and c_2 are constants that depend on the shape of removed fragments. In what follows we consider these constants to be independent of the fragment size and its number due to the scale similarity assumption. The constant c_1 is positive because the body volume decreases as we remove fragments. The constant c_2 can be either positive or negative.

It is necessary to express r_{n+1} in terms of V_n and S_n to close the recurrent scheme. After removal of many pieces, the size of next fragments is much smaller than typical sizes of the initial body. The structure of the remained body does not depend any more on the shape of the initial body and is determined by the shape of fragments. As n increases, the local structure of the body remains self-similar because the construction law is the same at all scales. The difference is only in the size of structure elements. From the similarity assumption it follows that the size of removed fragments is proportional to the size of these elements. The linear size of the structure element can be determined as the ratio of its volume to the surface area. Under this definition, the result does not depend on the quantity of elements considered, since both the volume and the surface area are proportional to this quantity. Thus we obtain the required expression:

$$r_{n+1} = c_3 V_n / S_n, \tag{2}$$

where c_3 is a positive constant dependent on the shape of removed fragments.

The resulting recurrent scheme gives the dependence of r on n and the distribution function:

$$r \propto n^{-\frac{c_2 c_3^2 + c_1 c_3^3}{3 c_2 c_3^2 + 2 c_1 c_3^3}} \tag{3}$$

$$f(r) = \left| \frac{dn}{dr} \right| \propto r^{-3 - \frac{1}{1 + c_1 c_3 / c_2}}. \tag{4}$$

In the process of calculation, we obtain a restriction on c_1, c_2 and c_3. From the restriction, it is follows that the exponent in this function falls into the interval from -4 to -1. All known experimental results are in this interval.

In the case of a N-dimensional body, similar calculations give the allowable range for the exponent between $-N-1$ and -1.

III. ANALOGY WITH FRACTALS

The above model of body decomposition resembles an algorithm of fractal construction and can be treated in terms of the fractal theory. It is difficult to present an example of a well-known fractal structure which is constructed by removal of fragments and for which the size of the fragment strictly decreases with its number. It is easier to design an example on the basis of a simple fractal like Sierpinski carpet [11]. In fractals of this kind, at every construction stage a number of equal-size fragments is removed. The process is characterized by two parameters: the ratio of linear sizes of fragments removed at consequent stages ($k_1 > 1$), and the ratio of numbers of fragments removed at consequent stages ($k_2 > 1$). We can obtain the dependence of r on n and the distribution function:

$$n \propto r^{-\log_{k_1} k_2} = r^{-D}, \tag{5}$$

$$f(r) = \left| \frac{dn}{dr} \right| \propto r^{-1-D}, \tag{6}$$

where D is the so-called fractal dimension [11].

In Ref. [4], a fractal structure of separate dust particles falling on a substrate has been detected. The fractal dimension of particles was measured to be 2.2, that corresponds to the exponent -3.2 in the dust size distribution function. However, parts of the dust particles cover each other if watched on the substrate, and the overlapping can change the measured size distribution.

IV. ADDITIONAL ASSUMPTION

The combination of constants in the expression (4) can be interpreted in this way:

$$\frac{c_1 c_3}{c_2} = -\frac{\Delta V_n}{\Delta S_n} / \frac{V_n}{S_n}, \tag{7}$$

where $\Delta V_n = V_{n+1} - V_n$ and $\Delta S_n = S_{n+1} - S_n$. If V_n^* and S_n^* are the area and the volume of the n-th fragment, then $\Delta V_n = V_n^*$ and $\Delta S_n \le S_n^*$. Hence it follows

$$\left| \frac{c_1 c_3}{c_2} \right| \ge \left| \frac{V_n^*}{S_n^*} / \frac{V_n}{S_n} \right|. \tag{8}$$

It is reasonable to assume that the shape of each fragment is more regular than the shape of the remained

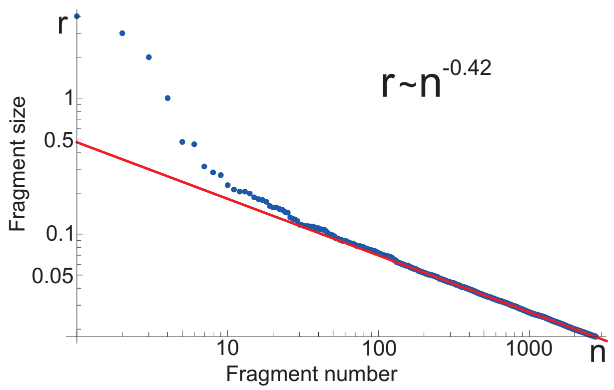

Figure 1: The dependence of ball radius on its number in the numerical simulation

body. In this case, the right-hand side of the inequality (8) is much greater than unity, as is the left-hand side. Consequently, the exponent in the size distribution function (4) differs from -3 by about the inverse large parameter (7). In the case of N-dimensional body fragmentation, similar calculations give the exponent close to $-N$.

V. NUMERICAL SIMULATION

The exponent in the distribution (4) equals -3 only in the case of infinite parameter (7). The expression (7) can be unlimited only if the c_2 is close to zero. It is possible if about half of the fragment surface coincides with the surface of the body from which it is extracted. We will try to determine numerically how close to -3 may be the distribution exponent, if the whole surface area of the fragment adds itself to the remained body.

Ratio c_1/c_2 is maximum for a ball, as a body with a maximum ratio of volume to surface area. The assumption about the fragment shape is appropriate because the shape of small duct particles on the substrate is close to circle [2]. The largest ball should be removed from the body for maximization of c_3. The result of numerical simulation of the case is on the Fig. 1. The corresponding distribution function has been found:

$$f(r) = \left|\frac{dn}{dr}\right| \approx r^{-3.4}. \qquad (9)$$

The result exponent in the distribution equals -3.4. The difference with -3 can be interpreted as a small parameter.

VI. CONCLUSION

We show that the scale similarity of fragmentation laws causes the linear size distribution of fragments to be the power function with the exponent between -4 and -1. The power exponent can be expressed in terms of the fractal dimension. The additional assumption of fragment shape regularity allows us to show that this exponent is close to -3. This value falls into the range of experimental observations. Thus, the brittle destruction can be responsible for the observed dust size distributions. The calculated exponent for the case of maximum ball removing equals -3.4.

The basic feature of the introduced model is that it explains identical results for dust size distributions for such different materials as graphite and tungsten. The distribution law depends on the space dimension and the shape of fragments rather than on physical properties of materials.

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