MHD STABILITY OF A PLASMA IN A SYSTEM OF COUPLED AXISYMMETRIC ADIABATIC OPEN-ENDED TRAPS

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Conditions for convective plasma stability in a system of coupled axisymmetric open traps with sign-alternative curvature of magnetic field are analyzed both in the MHD model and the Kruskal - Obereman kinetic model. For a couple of nonparaxial simple mirror cell and a semicusp, the "radial' interval where a hollow plasma can be stable is determined, as well as the range in which the ratio of the pressures in component cells should lie. Both external and internal plasma boundaries are stable in accordance with the average minB principle, provided that the pressure profiles in the cells are made consistent. The plasma compressibility plays an essential role. The stability of the cells against the global mode (as in the Ryutov – Stupakov trap) is sufficient but not necessary for stabilizing the chain. For the couple under consideration, the stability margin is not small.

I. INTRODUCTION

The convective (flute) MHD instability of a plasma in a simple axisymmetric open mirror trap [1] can be suppressed by connecting this trap to an adiabatic cell in which the magnetic field curvature has opposite sign, see, e.g., [2]. In particular, stabilization of a hollow plasma is possible in a couple of the simplest elements: a nonparaxial mirror trap and the adiabatic region of a semicusp. In this case, sufficient conditions for stability relatively any flute displacement are the following [3]: there exists a "stability ring" in the simple mirror [4] which is stable against global mode and the plasma in two cells occupies only this flux layer, the ratio of plasma pressures in the cells being near some fixed level.

Recently [5] we have established that stability conditions are more soft. The existence of the Ryutov – Stupakov stability ring [4] is not necessary. The plasma layer can include a significant fraction of the magnetic flux through the system, and the relative width of the allowed range of the ratio of pressures in the cells is not small. In the paper presented we formulate such conditions for the case of sharp plasma boundaries using the MHD model and the Kruskal – Oberman kinetic model and demonstrate stability in a specific configuration.

II. HOLLOW PLASMA STABILITY IN THE MHD MODEL

The configuration of interest is shown in Fig.1. Let mirror ratios be large. In this case the pressures in the cells 1,2 can be almost isotropic ($p_{\parallel 1} \approx p_{\perp 1} = p_1$, $p_{\parallel 2} \approx p_{\perp 2} = p_2$), though the levels p_1, p_2 are different (a modified MHD model [6]).

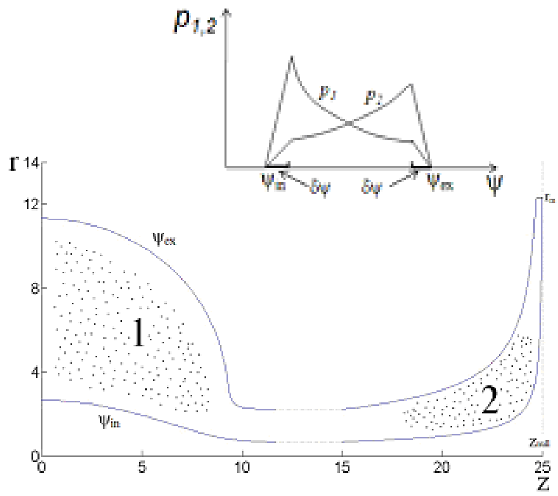

Fig.1. Field configuration and plasma pressure profiles

The number of transit particles is low, but we assume that its density is enough to provide (together with a cold linking plasma, if present) a high longitudinal conductivity, so that a flute is common to the cells. Contribution of these populations to the potential energy of a perturbation may be neglected. At $\beta << 1$ a sharp boundary is stable provided (see [1])

$$\sum_{1,2} \int \frac{\partial p}{\partial \psi} \frac{\partial B^{-2}}{\partial \psi} d\chi > 0 . \qquad (1)$$

Let the pressure $p_2 |_{\psi=\psi_{ex}-\delta\psi}$ be high enough to make the external plasma boundary in the couple stable (to satisfy inequality (1) at this boundary) providing some little margin. Let the pressure profiles in the layer $\psi_{in} + \delta\psi < \psi < \psi_{ex} - \delta\psi$ be $p_1 = P_1 U_1^{-5/3}$, $p_2 = P_2 U_2^{-5/3}$, where $P_{1,2}$ are constants, $U_{1,2} = \int_{1,2} B^{-2} d\chi$, χ is the longitudinal coordinate, $\nabla\chi = \mathbf{B}$. According to the MHD model this layer is marginally stable [7, 8]. The jump of the pressure p_1 at the internal boundary $\psi \approx \psi_{in}$ is high while the pressure p_2 jump is relatively low (see Fig.1). Since the field curvature on the surface $\psi = \psi_{in}$ in the cell 1 is favourable it is possible to stabilize not only the external boundary in the system but the internal boundary as well. Stability of both external and internal edges takes place if

$$\frac{dX^{MHD}}{d\psi} < 0, \quad X^{MHD}(\psi) \equiv \frac{(U_1^{-5/3} dU_1 / d\psi)}{(-U_2^{-5/3} dU_2 / d\psi)} \quad (2)$$

in the interval (ψ_{in}, ψ_{ex}) and

$$X^{MHD}(\psi_{ex}) < \frac{P_2}{P_1} < X^{MHD}(\psi_{in}). \quad (3)$$

We consider a conventional cusp geometry, $U_2 = (2b)^{-1} \ln(br_m^3 / 2\psi)$. When the left inequality (3) is violated it means that the pressure in the cusp is not enough to stabilize the convex external boundary in the simple mirror. If p_2 occurs too large (the right inequality is violated) then a source of instability is bad curvature at the internal boundary in the cusp cell.

The requirement (2) cannot be satisfied for an arbitrary mirror 1. We have to find a suitable geometry. The magnetic field structure can be calculated from equations [9]

$$\frac{\partial}{\partial\psi}\left(r\frac{\partial r}{\partial\psi}\right) + \frac{\partial^2 \ln r}{\partial\chi^2} = 0,$$

$$B = \left(r^2\left(\frac{\partial r}{\partial\psi}\right)^2 + \left(\frac{\partial r}{\partial\chi}\right)^2\right)^{-1/2} \quad (4)$$

where $r(\psi,\chi)$ is the distance from the axis to a point ψ,χ at a field line. Boundary conditions are $r|_{\psi=0} = 0$, $r|_{\psi=\psi_b} = r_b(\chi)$, where $r_b(\chi)$ is a function to be chosen. We take

$$r_b^2 = \psi_b^2 \chi_*^{-2} [\mathrm{ch}^{-2}(\chi/\chi_*) + \varepsilon\psi_*/\psi_b],$$

$$\psi_* = \psi_b/15, \; \varepsilon << 1.$$

Figure 2 shows the function $X^{MHD}(\psi)$ (2) calculated for such a choice. The region which should be occupied by a stable hollow plasma is $5.2 < \psi/\psi_* < 15$ (i.e., approximately, $7 < r|_{z=0}\, \chi_*/\psi_* < 16$).

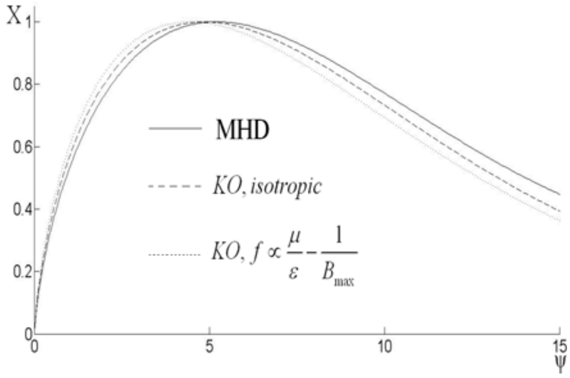

Fig.2. Normalized function $X(\psi)$

The relative width of the allowed range of the ratio p_2/p_1 is not small (~ 1). Note that, for the geometry considered, the Ryutov – Stupakov ring is $\psi/\psi_* > 10$. We can keep stability when put all the plasma outside this region, e.g., $\psi_{in} = 5.2$, $\psi_{ex} = 10$. The existence of such a ring is not necessary for plasma stability in the coupled cells.

III. STABILITY IN THE KRUSKAL – OBERMAN MODEL

In the Kruskal – Oberman model [10, 11] the pressure decrease index, $|d \ln p_1 / d\psi|$, for the marginally stable profile is greater than that in the MHD model, so stability conditions are more soft. These conditions can be received by replacing $X^{MHD}(\psi)$ in (2) with $X^{KO}(\psi)$,

$$X^{KO} = A\exp\left(-\int_{\psi_{in}}^{\psi_{ex}} \frac{S}{A} d\psi\right)\left(-U_2^{-5/3} \frac{dU_2}{d\psi}\right)^{-1}. \quad (5)$$

Here

$$A(\psi) = \int_{1/B_{max}}^{1/B_{min}} hG d\lambda,$$

$$S(\psi) = \int_{1/B_{max}}^{1/B_{min}} \left[h\frac{\partial G}{\partial\psi} + \frac{h^2}{T}\left(\frac{5}{2}G + \lambda\frac{\partial G}{\partial\lambda}\right)\right] d\lambda,$$

$$h(\psi,\lambda) = 2\frac{\partial}{\partial\psi}[\int B^{-1}(1-\lambda B)^{1/2} d\chi],$$

$T = \int B^{-1}(1-\lambda B)^{-1/2} d\chi$, $\lambda = \mu/\varepsilon$, μ is the magnetic invariant, $\varepsilon -$ the energy, the particle distribution in the cell 1 is $f_1 = F(\varepsilon)G(\mu/\varepsilon,\psi)u(\psi)$. For simplicity we use the MHD model to calculate the contribution of a cusp to the perturbation potential energy. The function $X^{KO}(\psi)$ for the case of isotropic pressure, $G = 1$, is shown in Fig.2, dashed curve. If the hot particles distribution is anisotropic so that $p_{\perp} > p_{\parallel}$ then the region where $dX^{KO}/d\psi < 0$ is still wider and the ratio $X^{KO}_{\max}/X^{KO}_{\min}$ is some more than in the case of isotropic pressure (see Fig.2, point line).

IV. CONCLUSIONS

So we have shown that, for a hollow plasma, stability conditions can be ensured in a combination of two elementary cells - a simple nonparaxial cell and the adiabatic region of a semicusp – provided that the pressure profiles p_1, p_2 are made consistent.

It is possible to shift stability margin from one boundary to another by varying the ratio p_2/p_1. The margin is not small: a characteristic scale of the average magnetic well is of the order of r. This margin can be used to stabilize an additional trap.

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