TAIL-WAVING SYSTEM FOR ACTIVE FEEDBACK STABILIZATION OF FLUTE MODES IN OPEN TRAPS

A. D. Beklemishev

Budker Institute of Nuclear Physics and Novosibirsk State University,
Prospekt Lavrent'eva 11, Novosibirsk 630090, Russia; beklemishev@inp.nsk.su

Feedback control is routinely used in modern plasma traps for adjusting plasma equilibrium on the transport time scale. Some intrinsic properties of magnetic mirrors make it possible to employ feedback control for stabilization of flute modes as well. Purely electromagnetic plasma-control system that is independent of line-tying or plasma conductivity to the end-plates is proposed. The system adds transverse flexibility to the plasma column, so that any growing perturbation can be deformed to become anti-ballooning. Anti-ballooning form means reduced flute amplitude in bad-curvature regions and enhanced amplitude in expanders or other traditional stabilizers, so that energy of the perturbation becomes positive and the mode is suppressed. Detailed analysis shows that transverse flexibility (or tail-waving) of the discharge can be employed for feedback stabilization even without good-curvature regions. The only requirement is that the discharge inertia (field-weighted plasma density) and the pressure-weighted field curvature are differently distributed along the discharge. If based on inertia, the stabilization mechanism resembles the rope-walker act. Estimates show that the power cost of such stabilization is reasonable and scales inversely with the trap length.

I. INTRODUCTION

Classic axially symmetric mirrors are inherently unstable to flute modes. There are many schemes proposed for their passive stabilization, but they usually carry a price in terms of the power cost or loss of axial symmetry. It is possible to retain simple geometry of the magnetic field with the use of active feedback stabilization of flute modes.

Negative feedback principle means that the plasma diagnostic should detect the development of perturbation and then, based on the diagnostic data, the control system should suppress the perturbation. However, any real diagnostic has finite sensitivity, while the control system is discrete and has a finite response time. Fortunately, strong magnetic field and relatively low curvature of field lines in mirror machines makes the growth rate of flute modes so low that the feedback stabilization in mirrors is technically feasible.

Feedback systems have been successfully employed for stabilization of interchange and ion cyclotron flute modes in open trap experiments in the '70s [1]. However, there was significant progress in plasma parameters since then and the tested approaches are now inapplicable. For example, the Arsenin & Chuyanov stabilization system of OGRA-III worked at very low densities, $10^8...10^{11} cm^{-3}$. The mainstream control mechanism for open traps that was tested [2] and explored theoretically for fusion-grade systems [3] is the axial feedback. It is based on unique property of open traps, namely, the possibility to change the plasma potential by biasing segments of end-plates. This process is similar to the passive line-tying stabilization mechanism and has the same fundamental drawback: once the axial confinement of plasma improves, its conductivity to the end-plates degrades. Thus the axial feedback requires cold and dense plasma shell surrounding and anchoring the plasma core[3]. Such setup is possible but not always convenient, in particular, it contradicts the "detached plasma" concept.

In contrast to the electrostatic "axial" scheme, electromagnetic feedback systems should be operable even with minimized electrical contact between the plasma and external electrodes, thus allowing good axial confinement. Two such schemes are mentioned in the early review article [1]. They have their limitations and drawbacks that make them hardly suitable for long high-beta devices, such as fusion-class open traps are expected to be. The first scheme is based on Ioffe rods, and the drawback is that the effect is nonlinear (the controlling current scales as a square root of mode amplitude) and hence the power consumption and the perturbation of equilibrium are great. The second scheme is applicable only in short non-paraxial mirror cells due to coil-placement requirements. Later theoretical work produced schemes with ponderomotive stabilization [4], and with magnetic stabilization of the resistive-wall mode (in this case the main stabilization is provided by the conducting plasma shell) [5].

This paper presents an alternative method of magnetic control of low-frequency flute perturbations in open traps. The magnetic system is designed to provide transverse flexibility to the discharge (Fig.1,2).

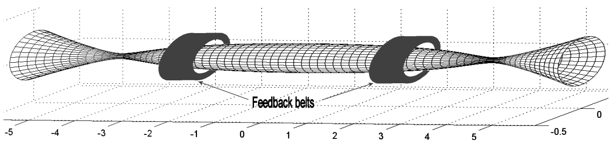

Figure 1: The control system consists of two symmetrically placed belts, each containing a number of coils for production of multipole transverse fields.

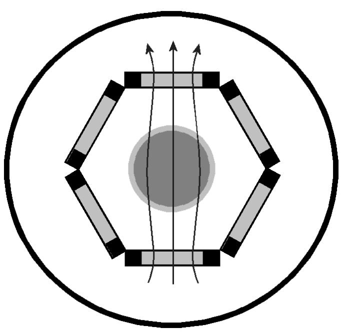

Figure 2: Schematic cross-section of a control belt.

It tries to force any developing flute mode to become strongly anti-ballooning in form, thus greatly enhancing influence of favorable-curvature parts (such as expanders), while minimizing (or even changing sign of) the contribution from flute parts with unfavorable curvature. In general its operation resembles balancing act by tail waving (see Fig.3), hence the title. In presence of line-tying each "tail's end" is also resistively anchored at the end-plate.

II. REDUCED-MHD DESCRIPTION

For description of a long and thin plasma column in a strong magnetic field we use the reduced-MHD approach developed by Kadomtsev & Pogutse[6]. Starting from the quasineutrality condition in the form $\mathrm{div} j_\parallel \mathbf{b} + \mathrm{div}\mathbf{j}_\perp = 0$, and obtaining the transverse component of the plasma current, $\mathbf{j}_\perp$, from the plasma momentum equation, it is straightforward to get

$$(\mathbf{B}\nabla)\,\frac{j_\parallel}{B} + c\nabla p\cdot\nabla\times\frac{\mathbf{b}}{B} - c\frac{\mathbf{b}}{B}\cdot\nabla\times\rho\frac{d\mathbf{v}}{dt} = 0. \quad (1)$$

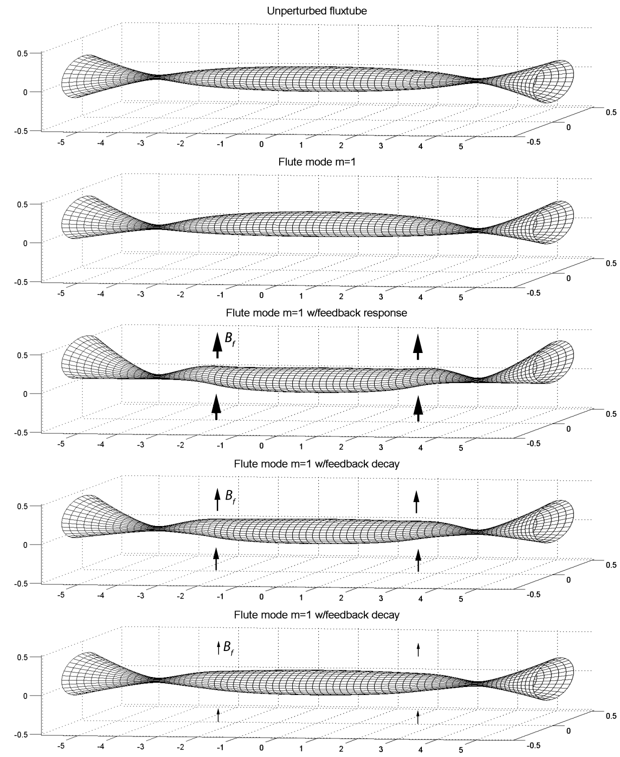

Figure 3: Schematic operation sequence (from top to bottom): [1-2]: The unstable flute mode $m = 1$ causes transverse plasma displacement, which is inversely proportional to the local magnetic field. [3]: Once the feedback is turned on, it causes the "tail swing" in the direction of the initial flute displacement. Due to inertia it also retracts the central part of the discharge. Displacement becomes *anti-ballooning*. [4-5]: The flute starts moving back. The feedback field is being reduced proportionally.

Instead of the $E \times B$ drift velocity, $\mathbf{v} = c\mathbf{b}\times\nabla\varphi/B$, where φ is the electrostatic potential, in the linear approximation it is more convenient to use the plasma displacement

$$\xi = \frac{\mathbf{b}}{B}\times\nabla\Phi, \quad (2)$$

where $\Phi = c\int\varphi dt$. Since the plasma velocity in our case can be taken as purely transversal and nearly incompressible due to small curvature, the perturbation of pressure is convective, $\tilde{p} \approx -\xi\nabla p_0$. The perturbed parallel current density can be expressed via the perturbation of the magnetic field, which, in turn, in the reduced MHD scheme is represented by the single (parallel) component of the vector potential. Then $\tilde{j}_\parallel = -\frac{c}{4\pi}\Delta_\perp A_\parallel$. After linearization and substitutions the quasineutrality condition (1) becomes

$$(\mathbf{B}\nabla)\,\frac{1}{B}\Delta_\perp A_\parallel + \nabla_\perp c_A^{-2}\nabla_\perp\ddot{\Phi} + K\nabla_y^2\Phi = 0, \quad (3)$$

where $K = 8\pi p_0' \kappa / B^2$ is the pressure-weighted normalized field curvature, $\kappa = \mathbf{e}_r \left(\mathbf{b}\nabla\right)\mathbf{b}$; ∇_y represents derivative in the direction of the binormal to the magnetic field, and c_A is the Alfvén speed. The closure condition for equation (3) is the parallel component of the Ohm's law, which for ideal conductivity yields

$$A_\parallel = -\left(\mathbf{b}\nabla\right)\Phi. \tag{4}$$

Now we are ready for introduction of the externally induced (by the feedback coils) small transverse magnetic field. Since it is transversal, it can also be described by the single component of the vector potential, $A_{\parallel f}$. However, it is the vacuum field, and thus requires no parallel current, $\Delta_\perp A_{\parallel f} = 0$.Still, its penetration into the plasma column causes plasma displacement due to induction fields, in accordance with the Ohm's law,

$$\Phi_f = -\int A_{\parallel f} d\ell, \tag{5}$$

where the integration is performed along field lines, and Φ_f is thus defined up to an arbirary field-line constant, which can be set by zero average. The displacement of plasma equilibrium position, ξ_f, caused by the feedback field, $A_{\parallel f}$, is then described by the drift equation, (2).

Representing the total perturbed field as a sum of the mode field and the feedback field, we can rewrite equation (3) as

$$\left(\mathbf{B}\nabla\right)\frac{1}{B}\Delta_\perp\frac{1}{B}\left(\mathbf{B}\nabla\right)\Phi - \nabla_\perp c_A^{-2}\nabla_\perp\ddot{\Phi} + K\nabla_y^2\Phi = \\ = \nabla_\perp c_A^{-2}\nabla_\perp\ddot{\Phi}_f - K\nabla_y^2\Phi_f. \tag{6}$$

The left-hand side of Eq.(6) is capable of describing Alfvén waves, interchange and ballooning modes, as its terms describe the field-line tension, plasma inertia, and the curvature drive of the interchange instability, respectively. The right-hand side represents the inertial and the curvature-caused driving forces due to the externally applied transverse magnetic field.

The usual way to restrain consideration to the low-frequency flute-modes only, is to integrate Eq.(6) along a field line and assume electrostatic approximation, $\Phi = const$ on each field line. Then the highest-order term (the field-line tension) vanishes, i.e., is expressed via current densities into the end-plates:

$$\left\langle K\nabla_y^2\right\rangle\Phi - \left\langle\nabla_\perp c_A^{-2}\nabla_\perp\right\rangle\ddot{\Phi} = \frac{4\pi}{cL}\left(\frac{j_{\parallel a}}{B_a} - \frac{j_{\parallel b}}{B_b}\right) + \\ + \left\langle\nabla_\perp c_A^{-2}\nabla_\perp\ddot{\Phi}_f\right\rangle - \left\langle K\nabla_y^2\Phi_f\right\rangle. \tag{7}$$

Here the angular brackets represent weighted field-line averages as $\langle f\rangle = L^{-1}\int(f/B)d\ell$, while indices a and b stand for values at opposite end-plates. Thus the first term in the right-hand side represents, if non-zero, the line-tying effects. Note that Φ_f is, by design of the control coils and Eq.(5), a non-constant function on a field line that can be optimized for a particular trap.

Equation (7) is the final one. If the source terms in the right-hand side are zero, it describes the ideal flute instability, or the resistive wall mode if the line-tying term is taken into account. It is also quite clear that by choosing a suitable amplitude and sign of $\Phi_f \propto \Phi$ it is possible to compensate or overcompensate the curvature drive:

$$\left\langle\nabla_\perp c_A^{-2}\nabla_\perp\ddot{\Phi}_f\right\rangle - \left\langle K\nabla_y^2\Phi_f\right\rangle > \left\langle K\nabla_y^2\right\rangle\Phi.$$

In general the inertial and the curvature source-terms are comparable to each other and to the left-hand side terms. Their relative value can be adjusted by particular placements of control coils, which coincide with nodes of $\Phi_f(\ell)$. Due to quite different parameter scalings of these terms it is highly unlikely that they will vanish together (except for the case of homogeneous plasma in a uniform field).

The "anti-ballooning placement" of control coils is to place them near inflection points of the field lines, so that Φ_f would change sign together with the curvature, K, thus maximizing the integral $\left\langle K\nabla_y^2\Phi_f\right\rangle$. Such placement is also compatible with elimination of the inertial source term, if it is required for simplification of the response algorithm. If the inflection points are too close to mirrors, or the favorable-curvature regions are too small to matter, it is still possible to produce a stabilizing response. Indeed, placing Φ_f nodes in such a way as to eliminate inertia, in general we are still left with a small but sign-definite curvature source. It can be used for feedback stabilization just as with favorable-curvature stabilizers, the only difference being that the required amplification, Φ_f/Φ, in this case is larger than unity.

The "inertial placement" of control coils is to minimize the curvature source term, so that the plasma response will be proportional to the second time derivative of the control current.

III. DISCUSSION

Results of the previous section show that if we can control the transverse magnetic field on a given field line, we can achieve relative stability near it. However, our goal is a global plasma stability, that would require independent field control on each field line, which is impossible. In practice, it would be reasonable to achieve stabilization of low-m asimuthal modes with a single radial harmonic for each mode (due to $\Delta_\perp A_{\parallel f} = 0$). Such limited goal requires only a few external coils and should be easy to achieve. Stabilization of low-m asimuthal modes is of interest for open traps with hot ions, since in such plasmas the high-m modes are suppressed by the FLR effects and require no feedback.

Another interesting question concerns time-response of the feedback system. The real plasma behaves in a more complicated way than described by the presented simple MHD model. The plasma rotates in the ambipolar field and the mode frequencies are shifted due to the drift effects. Such coherent rotation of modes can be anticipated by introducing proper phase shifts into Fourier-analysed diagnostic data while generating the feedback response signal. However, there is a physical limit on how high the response frequency can get. In fact, an instantaneous (light-speed) local application of the transverse field to the plasma column would result in generation of Alfvén waves traveling from the coil position. We have successfully avoided their description in the model while deciding to consider "the slow flute modes only", i.e., setting $\Phi(\ell) = const$. This is possible only if the frequencies involved are far below the Alfvén frequency, $\omega \ll c_A/L$, since perturbations have to propagate along the field with Alfvén velocity in order to produce the desired shape of perturbation elsewhere along the flute mode. In particular, the growth rate of modes to be stabilized should be sufficiently small. From the growth-rate estimate for the ideal interchange, $\gamma \sim V_{Ti}/L$, it follows that $V_{Ti} \ll c_A$, i.e., $\beta \ll 1$ is required for applicability of localized feedback coils. Otherwise, a distributed (along the trap) system of coils is required. The distributed system should also be capable of stabilizing low-m ballooning modes.

Estimate of needed power. Unlike the Ioffe rods, the coils of the proposed system produce influence that is first order in the coil current. The required tail displacements produced by the coils depend on the sensitivity of the sensor system and the background level of fluctuations. Assume that each belt of the feedback system is $\ell_f = 50cm$ long, has a radius of $R = 20cm$, can create radial displacements of $\xi = 1cm$ in the midplane, and has a frequency of $\nu = 10kHz$.

To create displacement of the plasma by ξ, the field produced by the coils should be $B_f = B_0\xi/\ell_f$, where B_0 is the field in the midplane. This field is created in the volume of $\sim \pi R^2\ell$ and is renewed with the frequency ν. If $B_0 = 10^4 gauss$, then the needed feedback field is $B_f \approx 200 gauss$, and without recuperation

$$W = 2 \cdot \nu \cdot \frac{B_0^2}{8\pi} \cdot \pi R^2\xi^2/\ell_f \approx 200kW.$$

Note that $\nu \sim L^{-1}$ so that the power consumption scales as

$$W \propto \xi^2 L^{-1}\ell_f^{-1}!$$

IV. CONCLUSION

Purely electromagnetic plasma-control system that is independent of line-tying or plasma conductivity to the end-plates is proposed. It is intended for stabilization of flute modes and has the following features:

- The tail-swinging control system should be symmetric in z with placement of coil belts around bad-curvature regions, in order to impose the "anti-ballooning" modulation on flute modes;
- Since the plasma reacts to feedback with Alfvén retardation, at $\beta \sim 1$, when $\gamma \sim L/c_A$, the localized coil system will not work. Stabilization of ballooning instability requires a distributed system of coils;
- FLR effects are essential, as the control system can only influence a few azimuthal modes with only a single radial harmonic each;
- The system should be able to create displacements of order of the diagnostic sensitivity or the fluctuation level. The required power for GDT parameters seems reasonable.

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