INFLUENCE OF MAGNETIC FIELD DIRECTIONS INHOMOGENEITY ON LONGITUDINAL PROPAGATION OF WAVE BEAMS

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In present communication analytical and numerical investigations of electromagnetic waves propagation near electron cyclotron resonance surface are presented taking into account variation of both magnetic field and magnetic field direction. Differences from analytical solution for constant magnetic field direction are revealed.

I. INTRODUCTION

In axisymmetrical magnetic traps inhomogenity of magnetic field intensity and magnetic field direction are closely related to each other. These two types of inhomogeneities essentially change the structure of electromagnetic wave beams with frequencies of the order of electron cyclotron frequency. For example it was numerically shown [1] that inhomogeneity of magnetic field direction could be responsible for strong refraction of electromagnetic waves in axisymmetric magnetic traps when plasma density higher than the critical one. Such refraction decreases the heating efficiency for quasi-longitudinal launch of rf power. This effect is related to peculiarities of the dependence of wave's group velocity on the direction of magnetic field near electron cyclotron resonance (ECR) region.

In this paper we present analytical investigation of electromagnetic waves propagation near ECR region of axisymmetrical magnetic trap with taking into account inhomogeneity of both magnetic field value and of its direction. Results are compared with well known analytical solution of Hamiltonian equations for ray propagation near ECR region under assumption that the direction of magnetic field is constant [2] (Note, nevertheless, that such magnetic field does not satisfy the Maxwell equation $div\boldsymbol{B}=0$). Conditions for ray concentration near the trap axis in the vicinity of ECR region are discussed which assumes effective wave absorption in the trap.

II. ANALITICAL MODEL OF RAY TRASING NEAR ECR REGION IN PARAXIAM MAGNETIC FIELD

Magnetic field in axisymmetrical system outside the region of its sources is determined from Maxwell equations:

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(rB_r\right)=-\dfrac{\partial}{\partial z}\left(B_z\right) \\ \dfrac{\partial}{\partial z}\left(B_r\right)=\dfrac{\partial}{\partial r}\left(B_z\right) \end{cases} \quad (1).$$

Solution of these equations in paraxial approximation may be presented as:

$$\begin{cases} B_z=J_0\left(r\dfrac{\partial}{\partial z}\right)F(z) \\ B_r=-J_1\left(r\dfrac{\partial}{\partial z}\right)F(z) \end{cases},$$

where $B_z(0,z)=F(z)$ is an arbitrary function, $J_{0,1}$ are Bessel functions presented as power series. Near ECR-surface magnetic field takes the form:

$$F(z)=B_0\left(1+\frac{z}{L_1}\pm\frac{z^2}{L_2^2}\ldots\right),$$

$$B_z\approx B_0\left(1+\frac{z}{L_1}\mp\frac{r^2}{2L_2^2}\right),\ B_r\approx -B_0\frac{r}{2L_1}, \quad (2)$$

$$B\approx B_0\left(1+\frac{z}{L_1}+\xi\frac{r^2}{L_1^2}\right),$$

with $\xi=\left(\pm\dfrac{L_1^2}{2L_2^2}-\dfrac{1}{8}\right)$. In the expression for B we take into account first non-zero terms over both coordinates z and r. Correspondingly terms in power series for B_z and B_r resulting in the next order corrections into the value of B are omitted.

For description of wave propagation we use ray-tracing technique in the cold plasma approximation. Near ECR region it is convenient to use ray Hamiltonian in form which do not include terms which tend to infinity at cyclotron frequency in spite of the fact, that some of dielectric tensor components possess such property. For example, such ray Hamiltonian for near longitudinal propagation of ECR waves may be written as [1]:

$$H(r,z,N_r,N_z) = N_\perp^2 + 2\frac{\varepsilon_\parallel}{\varepsilon_-}\left(N_\parallel^2 - \varepsilon_-\right) = 0\,, \qquad (3)$$

where $\varepsilon_\parallel = 1-\upsilon;\ \varepsilon_- = 1 - \dfrac{\omega_P^2}{\omega(\omega-\omega_B)}$, $\upsilon = \dfrac{\omega_P^2}{\omega^2}$

$N_\perp = N_r\cos\alpha - N_z\sin\alpha$, $N_\parallel = N_r\sin\alpha + N_z\cos\alpha$,

$tg\alpha = B_r / B_z$.

It can be shown, that in the vicinity of ECR region for quasi-longitudinal propagation direction of group velocity is determined only by sign of $N_\perp$. Using this fact one can see that the ray geometry depends only on the sign of $\partial N_\perp / \partial\tau$,

$$\frac{\partial N_\perp}{\partial\tau} = -\frac{\omega_B N_\parallel^2}{\omega_P^2}\varepsilon_\parallel\left(\cos\alpha\frac{\partial}{\partial r} - \sin\alpha\frac{\partial}{\partial z}\right)\omega_B\,, \qquad (4)$$

or for paraxial approximation (2) on the sign of the expression

$$\varepsilon_\parallel\left(\cos\alpha\frac{\partial}{\partial r} - \sin\alpha\frac{\partial}{\partial z}\right)\frac{B}{B_0} \approx \varepsilon_\parallel\left(\xi - \frac{1}{2}\right)\frac{r}{L_1^2}\,. \qquad (5)$$

If $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) < 0$, rays are “attracting” to the axis of magnetic system. If $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) > 0$, rays are pushed out of the system in radial direction. So, the condition

$$\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) < 0 \qquad (6)$$

may be considered as a condition of effective wave absorption near the trap axis.

In case $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) \approx 0$ terms which were neglected in (2), become of importance. It can be shown that if these terms are taken into account, additional critical points are arriving. For plasma with density lower than the critical one they are stable nodes. For plasma with density higher than the critical one they are «saddle» points. These points divide rays tending to the axis of a system and rays escaping in radial direction. It differs significantly from what takes place in the approximation of constant magnetic field direction, where governing parameter instead of (6) was $\varepsilon_\parallel\xi$ [2]. For the real magnetic trap conditions [3], $\xi \le 1$, term $\frac{1}{2}$ can not be neglected, and inhomogeneity of magnetic field direction may essentially affect ray behavior (and therefore, the heating efficiency) as compared to the approximation of constant direction of magnetic field.

III. NUMERICAL MODELING

In Figs. 1-4 results of numerical ray tracing modeling near ECR surface are presented, for different parameters of magnetic field distribution (for modeling we use expression for magnetic field components including all terms of paraxial expansion), and different values of plasma density, which is assumed to be hmogeneous. Ray trajectories are shown by solid lines, ECR surface is denoted by the dashed line. Results of numerical simulations fully confirm the resume of analytical investigation. When $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) < 0$ rays tend to axis of magnetic system, for $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) > 0$ rays tends to infinity in radial direction, and in case $\varepsilon_\parallel\left(\xi - \frac{1}{2}\right) \approx 0$ additional critical points are presented.

In Fig.5 results of modeling under assumption of constant magnetic field direction, and distribution of magnetic filed strength like in case of Fig. 2 are presented. For constant magnetic field direction approximation ray trajectories are appreciably different as compared to the case of real magnetic field distribution.

a) b)

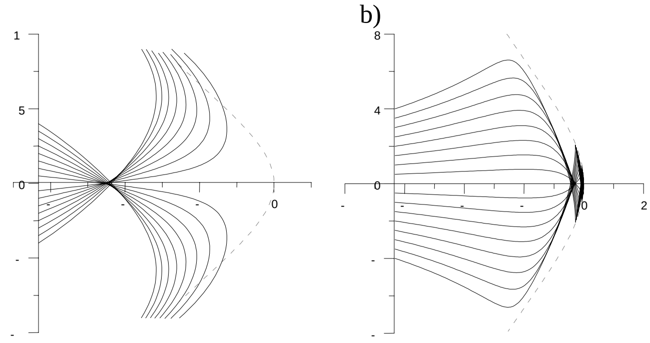

Fig. 1 Ray trajectories for $\xi = 1$, a) $\upsilon = 0.8$, b) $\upsilon = 2$

a) b)

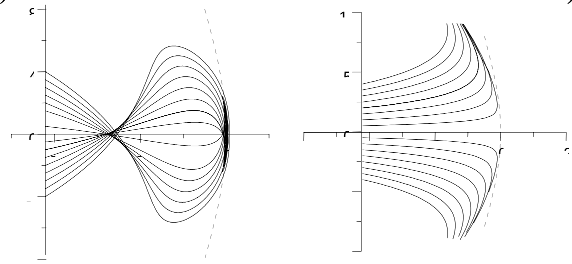

Fig. 2 Ray trajectories for $\xi = 0.3$, a) $\upsilon = 0.8$,b) $\upsilon = 2$

a) b)

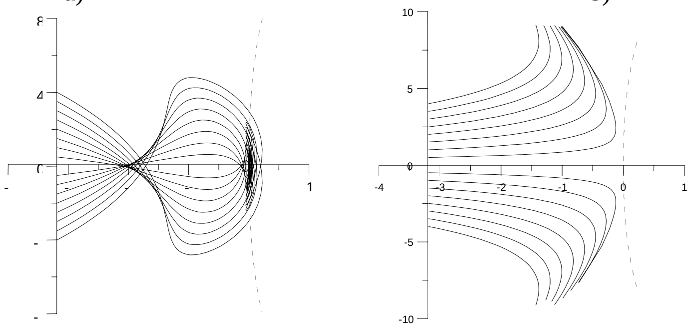

Fig. 3 Ray trajectories for $\xi = -0.3$, a) $\upsilon = 0.8$,b) $\upsilon = 2$

a) b)

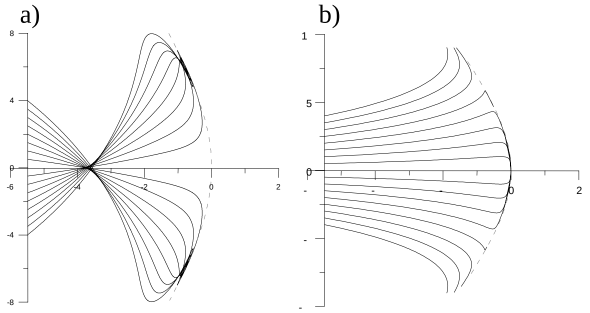

Fig. 4 Ray trajectories for $\xi = 0.5$, a) $\upsilon = 0.8$,b) $\upsilon = 2$

a) b)

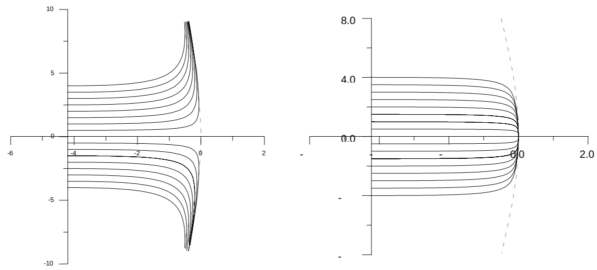

Fig. 5 Ray trajectories for $\mathbf{B} \parallel \mathbf{z}$, a) $\upsilon = 0.8$,b) $\upsilon = 2$

IV. CONCLUSIONS

In present communication analytical and numerical investigations of electromagnetic waves propagation near electron cyclotron resonance surface are presented taking into account variation of both magnetic field and magnetic field direction. Differences with analytical solution for constant magnetic field direction are demonstrated.

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