THEORETICAL FIELD AND COIL DESIGN FOR A SINGLE CELL MINIMUM-B MIRROR HYBRID REACTOR

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A vacuum magnetic field from a superconducting coil set for a single cell minimum-B mirror-based fission-fusion reactor is computed. The magnetic field is optimized for MHD stability, ellipticity and field smoothness. A recirculation region and wide magnetic expanders on both sides are provided to the central mirror cell. A coil set producing this field is computed which consists of circular and quadrupolar coils. Basic scaling assumptions are made for the coil dimensions, based on a maximum allowed current density of 1.5 kA/cm^2 for superconducting coils. Sufficient space is available for a fission mantle. The field produced by the coils is checked for MHD plasma stability and maximum ellipticity. The resulting confinement region is 25 m long with a 40 cm midplane plasma radius.

I. INTRODUCTION

The fusion-fission hybrid reactor, considered by Bethe[1] and others, is a combination of a fusion reactor and a fast fission reactor suggested for energy production, breeding of fissile material or transmutation of radioactive waste from fission plants. The fusion device is primarily used as a neutron source and the fission part surrounds the fusion reactor. For a power producing device, this reduces the requirements of the fusion Q factor by up to a factor of 140, which is far less demanding than that of a pure fusion reactor. The strong energy multiplication in the fission mantle opens the possibililty to use mirror machines as neutron sources. Mirror machines have several beneficial properties for fission-fusion devices[1], in particular a continuous mode of operation. The concept of such a machine is presented in Ref. [2], and in this work a preliminary version of a field and coil design for that device is presented. The concept of the Straight Field Line Mirror has been developed and adapted for the requirements that follow from space requirements for the coils and the fission mantle. A preliminary coil design that creates this vacuum field has been made that fulfills the spatial requirements of a fusion-fission device.

II. GEOMETRY OF THE DEVICE

The length of the confinement region has been set to 25 m. Beyond this, magnetic expanders with recirculation regions adds 6.25 m on each side, giving a total length of 37.5 m. The vacuum chamber has a radius of 0.9 m. Outside the vacuum chamber, the confinement region is covered by a 1.2 m thick fission mantle[2], where some space has been added for shielding of the coils. This gives a total radius of 2.1 m, and no coils are allowed inside this region. Also, some care has to be taken to the influx/outflux of the lead-bismuth eutectic that cools the fission mantle. Therefore, no circular coils are allowed between $|z| = 14.75$ m and $|z| = 16.10$ m, although quadrupolar coils parallel with the z axis are allowed. Coils are not allowed to intersect, and scaling assumptions has been made to approximate the size of the coils where existing coil systems[3, 4] has been used as a reference. A maximum current density of 1.5 kA/cm^2 is used and the structure material adds 20 % extra width.

The magnetic field strength at the center is set to 2 T and the device has a mirror ratio of four.

III. MAGNETIC FIELD OPTIMIZATION

The magnetic field is optimized on the following parameters:

1. Gross MHD stability. The criteria for MHD stability,

$$\frac{\partial}{\partial r_0}\int \frac{dl}{B} \leq 0 \qquad (1)$$

derived by Rosenbluth et al[5] has been used.

2. The flux tube ellipticity should be low for a number of reasons. A high ellipticity gives unpractical geometric properties and may for strongly elliptic regions give alpha particle gyro radii that are larger than the plasma minor radius. Also, highly elliptic plasmas will produce an angular-dependent neutron flux, which is unfavorable for the fission mantle.
3. Too strong gradients in the field components in the z direction have to be avoided, so that it is possible to create a practical coil set that produces those field components that lies outside the fission mantle. The thick fission mantle makes the coil design problem much harder since the distance between the coils and the plasma increases significantly.

These requirements are somewhat contradicting, and the task is to find a solution that is a suitable compromise for all requirements. A long-thin approximation has been used to express the magnetic field components to order o(λ^3). They are

$$B_x = \frac{x}{2}(g - \tilde{B}'), \quad B_y = -\frac{y}{2}(g + \tilde{B}'),$$
$$B_z = \tilde{B} + \frac{x^2}{4}\frac{d(g - \tilde{B}')}{dz} - \frac{y^2}{4}\frac{d(g + \tilde{B}')}{dz} \qquad (2, 3, 4)$$

where $\tilde{B}$ is the magnetic field on the z axis and g represents the quadrupolar field. The flux tube ellipticity is

$$\varepsilon_{ell}(z) = Max[e^{2G(z)}, e^{-2G(z)}], \quad G(z) = \int_0^z dz \frac{g}{2\tilde{B}} \quad (5, 6)$$

The stability criterion given in Eq. (1) is in flux coordinates

$$W_1 \cos^2\theta_0 + W_2 \sin^2\theta_0 \ \geq \ 0 \qquad (7)$$

where

$$W_{1,2}(z_{end}) = \int_{-z_{end}}^{z_{end}} \frac{dz}{\tilde{B}(z)} e^{2\int_0^z h_{1,2}(z')dz'} [h_{1,2}^2(z) + \frac{dh_{1,2}(z)}{dz}] \geq 0 \quad (8)$$

and

$$h_1(z) = \frac{g - \tilde{B}'}{2\tilde{B}}, \quad h_2(z) = -\frac{g + \tilde{B}'}{2\tilde{B}} \qquad (9, 10)$$

where θ_0 is the angular flux coordinate.

The task is now to optimize the functions g and $\tilde{B}$ to give a magnetic field with suitable properties. The method chosen is to represent the two functions g and $\tilde{B}$ with equidistant cubic clamped splines in the confinement region. For $|z| > z_{end}$, magnetic expanders (with favorable curvature) are expected to add to stability. Favorable stability properties in the region $|z| < z_{end}$ ought therefore to be sufficient to ensure gross MHD stability.

The functions $g(z)$ and $\tilde{B}(z)$ (with a spline representation) are found using a Nelder-Mead numerical local optimization method by minimizing a functional $f(\tilde{B}, g)$

$$f = K_{s2}\int_0^{z_{end}} W_1^2(z')dz' - K_{s1}\int_0^{z_{end}} W_1(z')dz' + K_{ell}\varepsilon(z_{\max})$$
$$-K_{aneg}K_{s1}\int_0^{z_{end}} W_1(z')H(-W_1(z'))dz' + K_{ripple}\int_0^{z_{end}} \left|\frac{dg(z')}{dz'}\right| dz' \qquad (11)$$

where K_{s2}, K_{s1}, K_{aneg}, K_{ell} and K_{ripple} are weight constants for stability discrepancies from marginal stability, stability, extra punishment for negative stability, ellipticity and the integrated absolute value of the derivative of $g(z)$ which suppresses ripple and peaked profiles. H denotes Heavysides step function. With a high weight on the ellipticity term K_{ell}, an average minimum-B field was found for a mirror ratio of four with an ellipticity of only 6.5 which may grow somewhat outside the confinement region. However, this field is unstable for smaller z_{end} and may not be overall stable. It also has very peaked magnetic field profiles. With a more moderate K_{ell} the final result was achieved. The fields, shown in Fig. 1, are smooth, the ellipticity is 17.5 and the integral $W_{1,2}(z)$ is positive for all z.

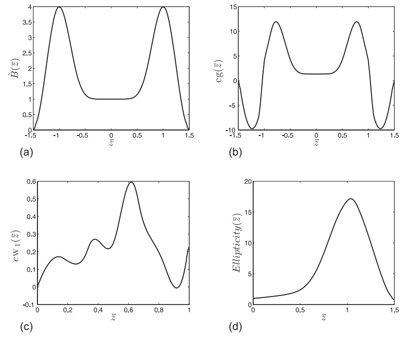

Fig. 1. The normalized field functions, the ellipticity and the stability integral $W_{1,2}(z_{end})$ for the optimized field.

IV. COILS

To reproduce the magnetic field selected in the previous section, a set of superconducting coils with specified currents are defined. The problem is simplified by using a filamentary set of line currents for each coil. The distance to the z axis from the coils are larger than in a fusion reactor (with the same fusion device) due to the fission mantle, which makes it harder to produce sharp relative derivatives $\partial/\partial z(\ln\tilde{B})$ and $\partial/\partial z(\ln g)$. It is in principal possible to create sharp relative derivatives by cancelling fields (with opposing currents in nearby coils), but such a method would be uneconomic and less accurate[6]. The choosen coil types are circular coils to produce the $\tilde{B}$ function and quadrupolar coils to produce the g function. Each coil type only contributes to one of the functions, and thereby the optimization problem is separated into two independent problems (except for space requirements). An array of quadrupolar coils is shown in Fig 2, where the currents go in opposite directions in adjacent straight bars and in adjacent quarter-circle segments.

Fig. 2. An array of 3 quadrupolar coils.

The optimizations are made using a local optimization method. The quadrupolar coils are placed inside the circular coils. The circular coils are parameterized using z position and current. The quadrupolar coils are parameterized using current and position. The optimization is made at the z axis. For both coil sets, functionals to minimize are specified which punish errors in the fields and current consumption, where the field errors outside the confinement region have much lower weight. Afterwards, the fields are calculated numerically, and stability and ellipticity are checked. In the confinement region, the maximum field errors are about 0.3 % in $\tilde{B}$ and about 1-1.5 % with a maximum of 4 % in g. In the expander region, the errors are larger, but the original field is not optimized here. The resulting coil set is shown in Fig. 3. and table 1-2.

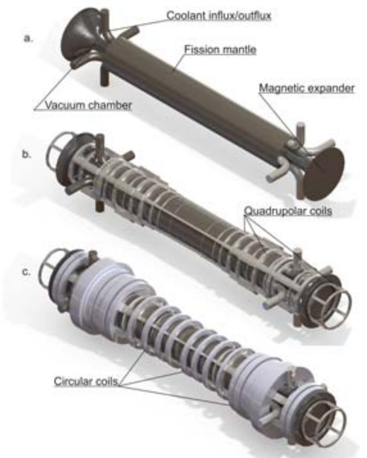

Fig. 3. The coil set that produces the field.

TABLE 1. Circular coil dimensions.

Inner radius (m)	z (m)	Coil Width (m)	Current (kA)
2.370	0.938	0.504	2651
2.370	2.794	0.483	2429
2.530	4.715	0.483	2428
2.790	6.909	0.360	1348
2.980	9.494	0.675	4743
2.980	10.500	0.826	7112
2.730	11.500	0.884	8131
2.730	12.650	1.092	12423
2.860	14.000	1.491	23158
2.860	17.300	0.591	-3634
2.740	18.125	0.633	-4176

TABLE 2. Quadrupolar coil dimensions.

End z(m)	Width (m)	Current (kA)	Closing Current (kA)
2.465	0.257	686.9	-17.5
4.340	0.263	721.8	-124.8
5.459	0.305	971.4	-453.8
6.924	0.425	1879.0	-1501.0
8.379	0.685	4881.1	-1329.1
9.619	0.851	7539.3	-234.5
10.901	0.877	8008.3	1988.7
12.498	0.622	4031.0	780.6
13.269	0.487	2469.9	2809.4
13.915	0.550	-3148.9	1424.7
16.251	0.759	-5998.3	-850.1
16.702	0.642	-4298.2	-71.3
17.700	0.631	-4155.6	-2077.8
19.375	0	0	-598.8
21.481	0.339	1197.6	598.8

V. CONCLUSIONS AND DISCUSSION

The magnetic field has been optimized for MHD stability, low ellipticity and low field gradients. The problem of designing the magnetic field for a fusion-fission reactor is different from the design problem of a pure fusion mirror reactor. The field is currently not optimized for neoclassical transport, since the confinement time is expected to be considerably lower than what would have been needed in a pure fusion reactor and therefore the major part of the plasma losses is expected to be at the mirror ends. It is crucial to find a magnetic configuration with low field gradients, since the coils that should produce the field are placed outside the thick fission mantle and are located far from the plasma. Therefore, the gradient steepness of g and $\tilde{B}$ along the z direction determines a minimum length of the machine. The initial idea was to use the SFLM field[7] concatenated with some other field near the mirror ends, but this field was hard to create with a coil set since it has too sharp gradients in the quadrupolar field. Therefore, the axisymmetric and quadrupolar field was modelled with a spline representation and optimized. The resulting optimized field has low z gradients, an ellipticity of 17.5 and the stability integral is positive. A preliminary coil set constituting of quadrupolar and circular coils has been computed. The coil set reproduces the field with maximum errors of 0.3 % in the $\tilde{B}$ function and 4 % in the g function. The resulting field has a positive stability integral everywhere. The coil set consists of 22 circular coils and an array of 27 quadrupolar coil segments.

Future work includes regarding effects of the pressure profile and ballooning modes.

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