The flute mode fluctuations and associated radial transport in the GAMMA10 A-divertor

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The flute instabilities were investigated in the GAMMA10 A-divertor magnetic field with help of computer simulations. The basic equations used in the simulation can be applied to only an axisymmetric system. So the high pressure in the remaining non-axisymmetric anchor cell, which is used for the flute mode stability, is taken into account by redefining the specific volume of a magnetic field line. It is found that the minimum-B mirror can stabilize a flute mode even in a divertor mirror cell, but its stabilizing effects are weaker. The radial transport accompanied by the flute instabilities in the GAMMA10 A-divertor is found to be rather smaller than that without a divertor mirror cell.

I. INTRODUCTION

The GAMMA10 tandem mirror is planning to install a magnetic divertor mirror cell (called GAMMA10 A-divertor) in it. A role of the GAMMA10 A-divertor is to evacuate the ions rapidly through x-point to the dipole region outside the divertor mirror cell. The divertor plate installed in the dipole region catches the ions lost from the divertor region, which is anticipated being the simulation of the divertor of a big torus.

The GAMMA10 A-divertor experiments have two operation modes. One is the steady state operation, where ions diffused from the core region of the central cell move along a magnetic field line to the x-point and then lost to the divertor plate continuously. Here the ion diffusion in the central cell is enhanced with the help of anisotropic electrostatic potential artificially created by ECRH and the non-axisymmetric magnetic field in the remaining anchor cell.

Another operation is the one like a disruption generating in a big torus, where the plasma is made lost rapidly from the central cell. This is one of the topic of this paper. The rapid loss of plasma is realized with help of flute instabilities.

The flute modes in the GAMMA10 A-divertor operation are stabilized by a high plasma pressure created in the remaining non-axisymmetric minimum-B anchor mirror cell. So the flute modes are destabilized when the high plasma pressure is destroyed by gas puffing in the anchor mirror cell. The other topic of this paper is to make clear the stabilizing effects by min.B mirror in a divertor mirror cell.

II. GAMMA10 A-DIVERTOR

We introduce the GAMMA10 A-divertor briefly in this section, which is illustrated in Fig.1. The vacuum chamber and coils of the present GAMMA10 are made use of as much as possible in Fig.1. The vacuum chamber surrounding the divertor and dipole regions, which will be newly built, has to be designed not to cross the floor.

Figure 1: The magnetic field in the GAMMA10 A-divertor.

Figure 2: Possible region of 1keV and 9keV ion motion. Here the ion guiding center is on the separatrix.

The ions in the dipole region outside the divertor are confined if its energy is less than 9eV. That is, the divertor plate can catch ions of energy less than 9eV. Figure 2 plots the possible region of ion motion, where
its guiding center is on the separatrix. The ion motion in an axisymmetric magnetic field is described by the equations

\[ M_{i} \dot{r} = -\frac{\partial \Phi}{\partial r}, \quad M_{i} \dot{z} = -\frac{\partial \Phi}{\partial z} \]  

(1)

Here the pseudo-potential \( \Phi \) is introduced.

\[ \Phi \equiv \frac{q^2}{2M_{i}c^2} \left( \frac{\psi - \psi_{0}}{r} \right)^2 \]  

(2)

The quantity \( \psi \) is the magnetic flux defined by \( B = \nabla \psi \times \nabla \theta \), and \( q, M_{i}, c \) are ion charge, mass and light speed. The ion velocity in the azimuthal direction is given by \( r\dot{\theta} = -q(\psi - \psi_{0})/(M_{i}cr) \). Figure 2 plots the equi-contour surface of 1keV and 9keV of \( \Phi \).

III. FLUTE STABILITY

The axisymmetric divertor mirror stabilizes the flute modes by means of plasma compressibility, i.e., \( pU^\gamma = \text{const.} \) is realized radially[1,2]. Here \( p \) is plasma pressure, \( U \) is the specific volume of a magnetic field line defined by \( U \equiv \int d\ell/B \), and \( \gamma \) is the specific heat index. The divertor mirror has \( B = 0 \) at x-point and \( U \) of the magnetic field line passing through x-point is infinitely large. Therefore, pressure \( p \) can be zero at the separatrix even if \( pU^\gamma = \text{const.} \) is satisfied.

However, the classical radial transport is large around x-point. The diffusion forms locally stable pressure profile (with \( \partial U/\partial \psi \) in the neighborhood of magnetic null and unstable pressure profile outside this area and so \( \partial pU^\gamma/\partial \psi = 0 \) is not satisfied around the plasma boundary, which can destabilize the flute modes. In order to understand the flute mode stability, we consider the tandem mirror stability criterion,

\[ \int \frac{(\hat{\rho}_{\perp} + \hat{\rho}_{||})\kappa_{\perp}}{B} d\ell \geq 0 \]  

(3)

Here a magnetic field line curvature \( \kappa \equiv \hat{e}_{\perp} \cdot \nabla \hat{e}_{\perp} = \kappa_{\perp} \nabla \psi + \kappa_{\theta} \nabla \theta \) and plasma pressures are represented by a separation of variables \( p_{\perp, \parallel}(\psi, B) = \hat{\rho}_{\perp, \parallel}(B) \nu(\psi) \).

Stability criterion (3) was used to the design of the GAMMA10 tandem mirror. Equation (3) does not assume a long thin plasma so that it can be applied to a divertor fat plasma. Noting that the vacuum magnetic field satisfies \( \nabla \times B = B \kappa \), Eq.(3) is rewritten as

\[ \int \frac{(\hat{\rho}_{\perp} + \hat{\rho}_{||})\kappa_{\perp}}{B} d\ell = -\frac{1}{2} \frac{\partial}{\partial \psi} \int \frac{(\hat{\rho}_{\perp} + \hat{\rho}_{||})}{B} d\ell \]  

(4)

The stability criterion (3), therefore, reduces to the classical stability criterion of flute modes \( \partial U/\partial \psi \leq 0 \) in the mirror cell[3], if the specific volume of a magnetic field line \( U \) is redefined as

\[ U \equiv \int \frac{(\hat{\rho}_{\perp} + \hat{\rho}_{||})}{B} d\ell \]  

(5)

Equation (5) takes into account the non-isotropic pressure effects in the stability criterion of the divertor mirror cell.

Figure 3: Axial profiles of pressure \( \hat{p} = \hat{p}_{\perp}(B) + \hat{p}_{\parallel}(B) \) in the GAMMA10 A-divertor.

The axial pressure profiles are plotted in Fig.3, where the pressures are given by

\[ \hat{p}(B) \equiv \hat{p}_{\perp}(B) + \hat{p}_{\parallel}(B) = \max \left( p_{A}(B_{m}^{2} - B_{z}^{2}) \right) \]  

(6)

Here \( p_{A} \) is the pressure at the anchor midplane; \( B_{z} \) is the magnetic field at the midplane on axis in anchor cell and \( B_{m} = 1.7B_{c} \), that is anchor pressure becomes unity at \( B(z) = B_{m} \). The pressure \( p \) in the other region is assumed to be unity.

Figure 4: Specific volume \( U \) for various \( p_{A} \).

Figure 4 plots the radial profiles of \( U \) for various anchor pressure \( p_{A} \) as a function of \( x \). Here \( x \) is the normalized radial coordinate defined by \( x = \sqrt{\psi/\psi_{0}} \). The quantity \( \psi_{0} \) is a coordinate of the outermost magnetic flux tube (separatrix), magnetic field lines on which pass through a magnetic null point (x-point). The appearance of \( \partial U/\partial \psi < 0 \) in the case of \( p_{A} \geq 12 \) indicate that the flute modes are stable in the core region when a high pressure is created in the anchor cell.

The nonlocal calculation of linear growth rate of flute instability is briefly shown. The radial profiles of density, temperature and vorticity are assumed to be

\[ \hat{\omega}_{0}(x) = +1.0, \quad \hat{\rho}(x) = 1.0 \]  

(7)

and

\[ \hat{T}_{0}(x) = \exp[-2x^2]U(x)^{-1} \]  

(8)

Here \( \hat{\rho}(x) = \rho(x)U(x), \hat{T}_{0}(x) = T(x)U^{-1}/M_{i} \), where \( \rho, T \) are ion mass density, temperature, which are described later as well as \( \omega_{0} \) in this paper.
Flute modes are stable in the case that $\hat{T}_0(x) = 1.0$ for all $U$ in Fig. 4. The linear growth rates of flute instability in the case of a slim $\hat{T}_0(x)$ of Eq. (8) are plotted by circles and in a fatter $\hat{T}_0(x)$ case of Eq. (9) are plotted by triangles in Fig. 5, where

$$\hat{T}_0(x) = \exp\{-0.5x^2\}U(x)^7 - 1$$  \hspace{1cm} (9)

Figure 5: Linear growth rates of flute instability for various $p_A$.

Figure 5 shows that the linear growth rate of $m = 1$ flute mode, where $m$ is the azimuthal mode number, depends on the $\hat{T}_0(x)$ [that is, $\hat{\rho}(x)$] as well as $p_A$.

IV. BASIC EQUATIONS

We describe the basic equations used in the simulation code briefly, where the detailed derivation of the basic equations, which are in the framework of MHD, is given in the references [4,5]. The coordinates used are $(x, \varphi, \zeta)$, where $\varphi$ is the azimuthal coordinate and $\zeta$ is a coordinate along a magnetic field line.

The specific dynamic vorticity $\hat{\omega}$, mass density $\hat{\rho}$, effective temperature $\hat{T}$, which are the quantities integrated over a unit magnetic flux tube, are defined as

$$\hat{\omega} \equiv \frac{\partial}{\partial \varphi} \left( \hat{\rho}(r^2) \frac{\partial \Phi}{\partial \varphi} \right) + \frac{\partial}{\partial x} \left( \hat{\rho} \left( \frac{1}{2} B^2 + \lambda^2 B^2 \right) \frac{\partial \Phi}{\partial x} \right),$$

$$\hat{\rho} \equiv \int \frac{\rho \hat{\rho}}{B} = \rho U,$$

$$\hat{T} \equiv (T_i + T_e)U^{-1}/M_i$$  \hspace{1cm} (10)

where $\rho$, $T_i$, $T_e$ are the local mass density, electron, ion temperatures, all of which are assumed to be constant along a magnetic field line, $\Phi$ is the electrostatic potential multiplied by light speed $c$ and $\lambda = U(\partial/\partial \varphi)(f_{(0)}^0 1/U J d\zeta) + (\nabla \psi \cdot \nabla \zeta)/(r^2 B^2 J)$, $J = \nabla \psi \times \nabla \varphi \cdot \nabla \zeta$. The field line averaged quantity $\langle A \rangle$ is defined by $\langle A \rangle \equiv \frac{1}{\hat{T}} \int d\ell \hat{A}$. The adiabatic velocity $v_\alpha$ of plasma has the form of

$$v_\alpha = -\nabla \Phi \times \frac{B}{B^2} + \frac{B}{B^2} \lambda \frac{\partial \Phi}{\partial \varphi},$$  \hspace{1cm} (11)

where the first and second terms in the right-hand side are the velocities across and along a magnetic field line, and the velocity satisfies $\nabla \cdot (v_\alpha / U) = 0$. The basic equations consist of four equations, that is, time evolutions of $\hat{\omega}(x, \varphi)$, $\hat{\rho}(x, \varphi)$, $\hat{T}(x, \varphi)$ and Poisson equation in Eq. (10) which determines the potential $\Phi(x, \varphi)$.

The basic equations can be applied under the assumption that the plasma pressure is below the instability threshold for Alfvén modes ($\beta < \beta_{cr} \sim 1$). And the high frequency stable collective degrees of freedom corresponding to magnetosonic, Alfvén, and longitudinal acoustic modes are excluded from the basic equations [4,5]. The effects of magnetic field line curvature on the flute interchange modes are included through the specific volume $U$.

The normalized quantities are used in the following, which are used in the simulation code. The normalized quantities $w_i$, $D$, $T$, which correspond to the quantities $\hat{\omega}$, $\hat{\rho}$, $\hat{T}$, respectively, are described by a sum of zeroth and perturbed quantities as, $w(x, \varphi) = w_0(x) + w_f(x, \varphi)$, $D(x, \varphi) = DE(x) + c^2 D\varphi(x, \varphi)$, $T(x, \varphi) = TE(x) + c^2 T\varphi(x, \varphi)$. Here $c^2$ is a small expansion parameter defined by $c^2 \equiv (\chi_M/b_{cr}M^2[2T_i/(T_i + T_e)]^2$, where $\chi_M = \sqrt{\gamma T_i/M}$, $b = \sqrt{\psi_0/B_M}$, and $\chi_M = [T_i/(M_i \omega_i \tau_i)]M$ which is related to the classical thermal diffusibility; $c^2 = 10^{-2}$ is assumed in the paper. The quantities $\omega_i$, $\tau_i$ are cyclotron frequency, ion coulomb collision time, respectively. The subscript $M$ means the quantity at the midplane on axis of the divertor mirror cell.

V. LINEAR ANALYSIS

The basic equations given in Ref. [4,5], which are used in the simulation, gives a set of equations for the nonlinear MHD convection. The linear analysis is helpful to understand those equations. The perturbed function $w_f$ is expanded in Fourier series as

$$w_f = \sum_m w_f(m) = \exp(-i\omega \tau + im \varphi) \cdot \hspace{1cm} (12)$$

Here $i \equiv \sqrt{-1}$, $\omega$ is a frequency of linear wave and $m$ is an azimuthal mode number. The remaining unknown functions $D_f$, $T_f$, $\Phi$ are expanded in the same way.

Because the functions $\Phi$, $w_f$, $D_f$, $T_f$, $U$ depend on $x$, the nonlocal treatment is required to obtain the dispersion equation.

$$\frac{\partial}{\partial x} \left( D_{Ef} \frac{\Phi(f)}{x} \right) - \frac{\partial}{\partial x} \left( \frac{D_{Ef}}{2x^2} \Phi(f) \frac{\Phi(f)}{x} \right) \left( \omega - \frac{m}{2x} \frac{\Phi(f)}{x} \right) - \frac{4x}{x} \left( f_3 + f_4 \right) m^2 + \frac{4xw(f)_{(m)}}{\Phi(f)_{(m)}} \Phi(f)_{(m)} = 0.$$  \hspace{1cm} (13)

Here $\Phi_0(x)$ is determined by

$$\frac{\partial}{\partial x} \left( D_{Ef} \frac{\Phi_0}{x} \right) = 4xw_{0}.$$  \hspace{1cm} (14)
and $f_1$, $f_2$, $f_3$, $f_4$ are the geometrical factor defined by $f_1 = \langle r^2 \rangle/b^2$, $f_2 = \langle r^4 \rangle/b^4$, $f_3 = (1/\sqrt{2})\langle B^2 \rangle b^2$, $f_4 = \langle \lambda^2 B^2 \rangle b^2$, where $b = \sqrt{\psi_0/B_M}$ and $B_M$ is the magnetic field at the divertor midplane, and the last term $w_f(m)/\Phi(m)$ in left-hand side of Eq.(13) is described as a function of zeroth-order quantities, which is written in Ref.[4].

Equation (13) is an eigen-value equation with eigen-function $\Phi(m)$ and eigen-value $\omega$, which can be solved with a boundary condition of $\Phi(m)(0) = 0$ at $x = 0$ and at $x = 1$. The solutions of Eq.(13) is plotted in Fig.5 for the zeroth quantities Eqs.(7), (8), (9). Equation (13) is used in next section in order to investigate the linear phase of the simulations.

IV. SIMULATION RESULTS

Boundary conditions adopted in the simulations are $\partial D(x = 1, \varphi)/\partial x = 0$, $\partial T(x = 1, \varphi)/\partial x = 0$, $\Phi(x = 1, \varphi) = 0$. The variable $w(x = 1, \varphi)$ is programmed so as to conserve $\int_0^1 x dx \int_0^{2\pi} dp w(x, \varphi)$ in time. Initial conditions are $D(x, \varphi) = 1$, $T(x, \varphi) = 1$, and $w(x, \varphi) = +1$. Those boundary and initial conditions are adopted in all simulations in this paper.

IV.A. The Case Unstable to a flute Mode

At first, the case of $p_A = 1$ is described, which corresponds to the case without high pressure in anchor mirror and so is unstable to a flute mode in Fig.4.

Figure 6 shows the linear phase for $p_A = 1$.

Figure 6 plots the time evolution of $\Phi(m)$ in the linear phase observed at $x = 1/2$. The $m = 1$ Fourier amplitude of $\Phi(m)$ grows in time, the growth rate of which agrees with the solid line in the figure. Solid line is the linear growth rate determined by the nonlocal linear analysis (13), where the radial profiles of $D_E(x)$, $T_E(x)$ and $\Phi_0(x)$ in Figs.7(a) and 7(b) are used.

Figures 7(c) plots the radial profile of $m = 1$ Fourier amplitude of $\Phi(m=1)$ observed in the simulation at the normalized time $t = 80$, and Fig.7(d) plots the eigen-function of $\Phi(m=1)$ obtained by Eq.(13). Both radial profiles agrees well with each other. So the simulation results at $t = 80$ are the linear growing phase.

The contour plots of $w_f$, $\Phi$, $D_f$, $T_f$ in Fig.8 show that $m = 1$ mode is dominant in all perturbations and those are localized around $x = 1/2$ at $t = 80$. Here $\Phi$ are the potential $\Phi$ subtracted $m = 0$ component.

The long-time behavior of Fourier amplitudes of $\Phi(m)$ is plotted in Fig.9. The flute instability saturates at $t = 100$ and after that the Fourier amplitudes of $\Phi(m)$ repeat the growing up and down in time. The long-time behavior of these Fourier amplitudes can be understood in the following consideration.

Figures 10(a)–(c) plot the time variations of $\Phi(x, \varphi)$, $D(x, \varphi)$, $T(x, \varphi)$ observed at $(x, \varphi) = (0, 0)$ and $(1/3, 0)$. Fourier amplitude of $\Phi(m)$ is plotted in Fig.10(d). There are several feature to be emphasized. A classical resistivity, a classical thermal diffusivity and a classical viscosity are included in the basic equations. It is seen that the amplitudes of $D(x, \varphi)$, $T(x, \varphi)$ decrease slowly in the classical diffusion process before $t = 100$ (in the linear growing phase of flute instability). Around
\(\tau = 100\) there is large decay of \(D(x = 1/3, \varphi = 0)\) and \(T(x = 1/3, \varphi = 0)\) accompanied with a rapid oscillation, at the time of which is just the same time as a flute instability saturates. The large decay accompanied with a rapid oscillation is observed repeatedly in time, which coincides with the time when the Fourier amplitude of \(\tilde{\Phi}\) has a maximum in Fig.10(d).

\[
\Phi_{(m)}^{(a)} \text{ at } x = 1/2
\]

Figure 9: Time variations of Fourier modes of \(\tilde{\Phi}\) at \(x = 1/2\) in a non-linear saturation phase.

\[
\Phi(x, \varphi), \ D(x, \varphi), \ T(x, \varphi) \text{ observed at } (x, \varphi) = (0, 0) \text{ (Blue lines)} \text{ and } (x, \varphi) = (1/3, 0) \text{ (Red lines)} \text{ and Fourier amplitude of } \Phi_{(m)} \text{ are plotted.}
\]

Figure 10: Time variations of \(\Phi(x, \varphi), \ D(x, \varphi), \ T(x, \varphi)\) observed at \((x, \varphi) = (0, 0)\) (Blue lines) and \((x, \varphi) = (1/3, 0)\) (Red lines) and Fourier amplitude of \(\Phi_{(m)}\) are plotted.

\[
\Phi(x, \varphi) \text{ at } x = 0 \text{ and } x = 1/3
\]

\[
D(x, \varphi) \text{ at } x = 0 \text{ and } x = 1/3
\]

\[
T(x, \varphi) \text{ at } x = 0 \text{ and } x = 1/3
\]

Figure 11: Radial profiles of: (a) \(D_E(x)\) and (b) \(T_E(x)\) at \(\tau = 100\). Radial diffusions of: (c) density \(\Gamma_\rho(x)\), and (d) temperature \(\Gamma_T(x)\) at \(\tau = 100\). Solid lines in (c) and (d) are the anomalous diffusion and dashed lines are classical diffusion.

\[
\text{IV.B. The Case Stable to a Flute Mode}
\]

In the case of \(p_A = 20\) there is a magnetic well in \(x < 0.45\) in Fig.4. The question is whether a flute mode is stabilized or not by a magnetic well.

\[
\text{Figure 12: (a) } D_E(x) \text{ and (b) } T_E(x) \text{ at } \tau = 100. \text{ (c) } \Gamma_\rho(x), \text{ and (d) } \Gamma_T(x) \text{ at } \tau = 130. \text{ Solid lines in (c) and (d) are the anomalous diffusion and dashed lines are classical diffusion.}
\]

\[
\text{Figure 13: Linear phase for } p_A = 20.
\]
The simulation is performed with the same boundary conditions and initial conditions as that in Sec.IV.A. Figure 13 plots the time evolution of $\Phi_m$ and Fig.14 plots the radial profiles of $D_E(x)$, $T_E(x)$ and $m = 1$ Fourier amplitude of $\Phi$ at the linearly growing phase $\tau = 180$. Although $D_E(x)T_E(x)$ is not const. in whole region, the radial profile $\Phi_m$ in Fig.14(c) is different from the eigen-function obtained by non-local linear theory. And the linear growth rate of $\Phi_m$ at $\tau = 180$ is quite different from the growth rate obtained by the linear theory (solid line in Fig.13). That is, this case of $p_A = 20$ is not unstable to a flute mode because there is no linear grow phase in the simulation.

Figure 15 plots the radial profile of $D_E(x)$ and $T_E(x)$ observed at $\tau = 320$. In the region $x \leq 0.45$, where $\partial U(x)/\partial x < 0$, the $D_E(x)T_E(x)$ has a Gaussian type of radial profile.

V. SUMMARY

We performed the flute mode simulations in order to research the flute instability, associated radial transport, and the effect of a magnetic well to the flute instability. It is found that the flute instability enters a nonlinear saturation phase by changing the unstable radial profile $D_E(x)T_E(x)$ to the stable radial profile after a linear phase. In the saturation phase the flute mode amplitudes repeat the grow up and down by causing the anomalous radial transport. The flute instability fortunately (unfortunately) is not so strong to destroy the plasma immediately.

Figure 16 plots the maximum $m = 1$ amplitude of $\Phi$ observed in the simulations. In the case of $p_A = 20$, the maximum amplitude of $\Phi$ is one order magnitude smaller than the other case of $p_A$. Minimum-B mirror is found to have a tendency of stabilizing the flute modes even with the divertor mirror cell. However even if there is a magnetic well around axis ($p_A = 12$), which is stable to flute modes in a long thin approximation, flute modes are unstable as long as the well is not so deep. Thus higher pressure in anchor mirror than the present GAMMA10 is required to stabilize the flute modes completely.

The linear growth rates agrees well between the simulation and non-local linear theory as shown in Fig.17.

References