SIMULATIONS OF TURBULENT PLASMA HEATING BY POWERFUL ELECTRON BEAMS

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Basic mechanisms of turbulent plasma heating by powerful electron beams are studied using numerical simulations. Both particle-in-cell and hybrid codes are used to investigate how beam-plasma instability evolves and saturates in the case of continuously injected electron beam. For sufficiently high plasma temperature beam driven turbulence is found to operate in the regime of the constant pump, when the saturation level of heating power is determined solely by beam nonlinearities.

I. INTRODUCTION

The aim of the paper is to study the nonlinear evolution of beam-plasma system in the case of continuously injected electron beam. The feature of this regime is that the parameters of beam driven turbulence can significantly evolve during injection. It means that at different stages of beam-plasma interaction the rate and efficiency of plasma heating are governed by different physical processes. Analytical treatment of such a problem is hardly tractable. For this reason, the full chronological scenario of collective beam relaxation in a plasma is investigated here by means of computer simulations.

We focus our attention on the study of the role played by beam nonlinearities at the stage of steady-state turbulence. An attempt to account for trapping effects in the beam relaxation problem was made in [1]. It was assumed that the power of beam energy losses saturates with the appearance of beam trapping, and the relaxation model based on this idea gave surprisingly good agreement with experimental results. Thus, as to provide an adequate simulation of beam excited plasma turbulence one should describe in details the nonlinear kinetic effects associated with beam trapping. In principle, the problem of beam relaxation in fully kinetic framework could be treated by PIC simulations, but, in practice, an accurate description of both beam and plasma species is a challenging task even in one-dimension. The problem is that high-frequency short-wavelength plasma oscillations should be resolved over the large scale sizes of real experiments ($\sim 0.1 - 1$ μs and ~ 1 m). However, if the idea of Ref. [1] is true, and the full energy flux pumping by an electron beam into a plasma does not really depend on the turbulence nature, a careful description of plasma turbulence seems to be less important than accurate simulation of beam kinetics. Thus, as to simulate real beam-plasma experiments over large spatial and temporal scales, we propose a simplified hybrid model in which the electron beam is simulated by individual particles, whereas plasma dynamics is governed by fluid equations in analogy with the Zakharov's approach [2]. In order to control the validity of such a simplified description, the results of the hybrid model are checked by more detailed PIC simulations [3].

II. HYBRID MODEL

In the standard approach based on the Zakharov equations [2], plasma turbulence is described by a slowly varying amplitude of high-frequency Langmuir oscillations and a low-frequency density perturbation. Averaged description of the high-frequency field, however, is not suited for investigation of the effects of nonlinear beam dynamics on plasma waves excitation. To incorporate these effects into a model we should abandon time averaging of fluid equations over the plasma period and describe the high-frequency plasma response using the complex field Ψ:

$$\frac{\partial \Psi}{\partial t} + i\omega_p(x)\Psi - \gamma_l \frac{\partial^2 \Psi}{\partial x^2} = -4\pi j_b. \qquad (1)$$

Here, the plasma frequency $\omega_p(x) = \sqrt{4\pi e^2 n(x)/m_e}$ is calculated from the local value of slowly varying ion density $n(x)$, e and m_e are the charge and the mass of electron, j_b is the current density of beam particles, and the term $\gamma_l \Psi''_{xx}$ is artificially included in the model to provide the energy sink for the short-wavelength Langmuir oscillations. Physically, the complex field

Ψ is interpreted in the following way. The real part is identified with the electric field of Langmuir waves $E = \mathrm{Re}\Psi$, the imaginary — is associated with the current density produced by plasma electrons quivering in the HF field $j_p = \omega_p \mathrm{Im}\Psi/4\pi$, and the square modulus determines the wave energy density $W = |\Psi|^2/8\pi$. In a uniform plasma without a beam, Eq. (1) describes decaying plasma oscillations with the dispersion relation $\omega_k = \omega_p - i\gamma_l k^2$. For sufficiently small values of γ_l, such a dissipation model is able to qualitatively reproduce natural situations when beam excited plasma waves appear to be free of damping and the short-wavelength part of the spectrum experiences strong dissipation. The beam of density n_b is represented here by a number of ideal charged planes, $j_b = -\sum_j \sigma_j v_j \delta(x - x_j(t))$, which move with the velocity $v_j(t) = \dot{x}_j$ and have the surface charge density $\sigma_j = e n_b v_j(0)\Delta t$. Excitation of plasma oscillations at the point x in this case looks like the sequence of momentary forces each of them makes a push of plasma electrons and results in the local electric field increment $4\pi\sigma_j$. The dynamics of beam particles under the excited field E is self-consistently described by the equation

$$\frac{d}{dt}\left(\frac{m_e \dot{x}_j}{\sqrt{1 - \dot{x}_j^2/c^2}}\right) = -eE(t, x_j), \qquad (2)$$

where c is the speed of light. In the problem of continuous beam injection, the trapping effects alone do not saturate wave energy growth. This role should be played by nonlinear processes in a plasma. In this model, the mechanism responsible for the energy transfer from the source to the dissipation region of the wavenumber spectrum is associated with the plasma nonlinearity $\delta n(x)\Psi$ describing high-frequency Langmuir waves scattering off plasma density fluctuations. Although we do not account for thermal corrections to Langmuir wave dispersion, Eq. (1) also includes the effect of modulational instability. In this case, however, the instability is driven by model dissipation, which tends to localize wave energy in regions of small density gradients.

As to describe the slow plasma response to the high-frequency fields, we take into account generation of plasma density depressions produced by the ponderomotive force as well as the increase in plasma temperature due to Langmuir wave dissipation:

$$\frac{\partial^2 n}{\partial t^2} - \gamma_s \frac{\partial}{\partial t}\frac{\partial^2 n}{\partial x^2} - \frac{1}{m_i}\frac{\partial^2}{\partial x^2}\left[nT + \frac{\langle |\Psi|^2 \rangle}{16\pi}\right] = 0, \qquad (3)$$

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} - \frac{2\langle Q \rangle}{3n}, \qquad (4)$$

where m_i is the ion mass. To control the level of short-wavelength ion-sound perturbations, we also include artificial dissipation with the rate $\gamma_s k^2$. The temperature of plasma electrons T is governed by the heat conduction equation with the thermal conductivity κ. The role of heat source in this equation is played by the dissipation power averaged over the plasma period

$$Q = \frac{\partial}{\partial t}\left(\frac{|\Psi|^2}{8\pi}\right) = \frac{\gamma_l}{8\pi}\left(\frac{\partial^2}{\partial x^2}|\Psi|^2 - 2\left|\frac{\partial \Psi}{\partial x}\right|^2\right). \qquad (5)$$

III. RESULTS OF SIMULATIONS

Let us consider nonlinear evolution of the beam-plasma system during long-time injection of the cold weak electron beam with the density $n_b/n = 6.25\cdot 10^{-4}$ and with the velocity $v_b = 0.9c$ into the uniform plasma with the density $n = 10^{15}\mathrm{cm}^{-3}$ and the initial electron temperature $T = 50$ eV.

III.A Dynamic stage

As it is shown in Fig. 1, at the initial stage of beam relaxation the hybrid and PIC simulations quantitatively agree with each other. On plots of both energy

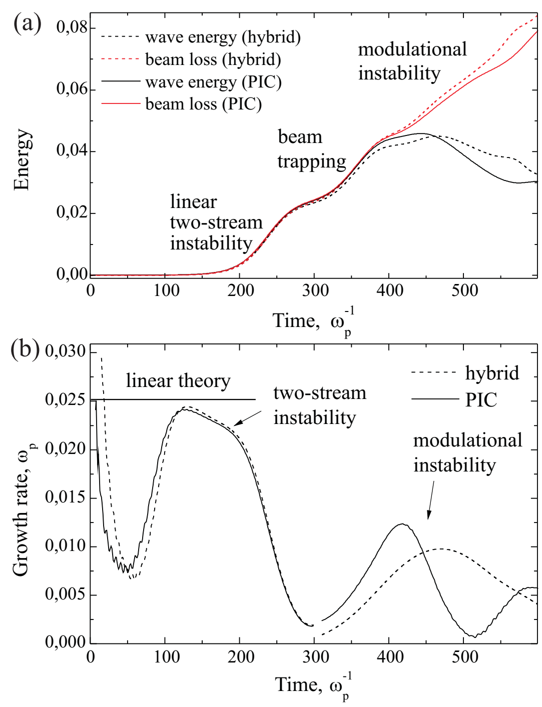

Figure 1: Dynamic stages of beam relaxation in different models: (a) dynamics of the energy lost by the beam and the wave energy; (b) growth rates of two-stream and modulational instabilities.

lost by the beam and plasma wave energy, the linear stage of two-stream instability is clearly observed. Energy of resonant waves at this stage grows almost exponentially, although the rate of this growth in the open system appears to differ slightly from the theoretical prediction calculated in the initial-value problem (Fig. 1b). Decreasing in the energy growth rate that follows afterwards is explained by trapping of beam electrons in the potential well produced by the resonant plasma wave. As a consequence of this nonlinear effect we observe localized wave packets, which have a similar form and location in different simulations (Fig. 2). In the open system, the occurrence of beam trapping

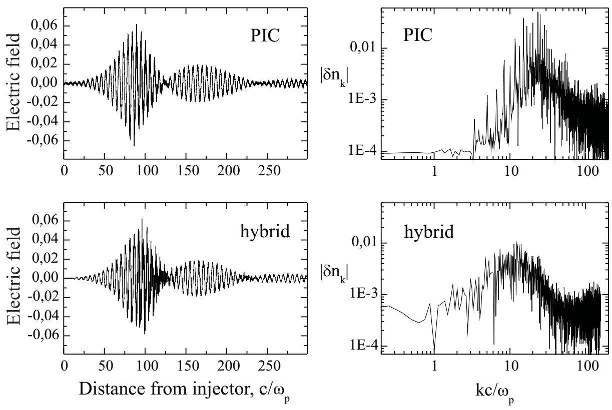

Figure 2: Electric fields in localized wave packets and k-spectra of density fluctuations at the moment of rapid growth of modulational instability $t = 450\omega_p^{-1}$.

effects does not result in saturation of resonant wave energy. It increases further, even if slower in comparison with the linear growth rate, up to the level when plasma nonlinearities come into force. By the time $t = 400\omega_p^{-1}$ the resonant wave energy exceeds not only the modulational threshold, but thermal plasma energy as well. For this reason modulational instability should play an important role in further evolution of the beam-plasma system. Fast growth of ion density fluctuations is really observed in both simulations. Although the hybrid model somehow distorts this nonlinear effect, the growth rate of modulational instability is found to be close to the result of PIC simulations (Fig. 1b). As Fig. 2 shows, the most unstable modulational perturbations have large wavenumbers ($k = 15\omega_p/c$ in the hybrid and $k = 23\omega_p/c$ in the PIC model). The feature of this case is that corresponding wavelengths are found to be close to either the Debye length r_D ($kr_D \sim 1/3$) or the length $r_E = eE/m_e\omega^2$ characterizing quiver motion of plasma electrons under the high-frequency field ($kr_E \sim 1$). It means that effects of finite electron temperature as well as strong electron nonlinearities should play an important role. Thus, in the case of the strong pump $W > nT$, modulational perturbations appear to be unstable within the region of strong dissipation, and no additional mechanism of the energy transfer is required.

III.B Modulational instability

The conclusion regarding short-wavelength character of the supposed modulational instability should be studied in more details. It is worthwhile to check

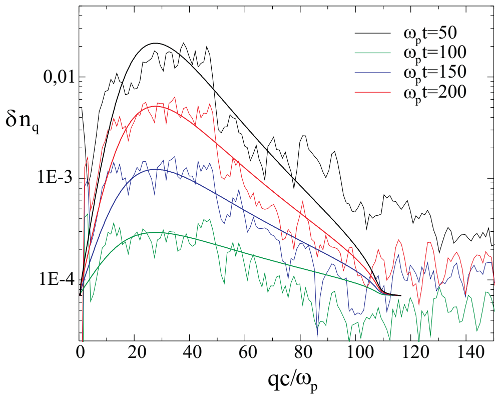

Figure 3: Comparison of theoretical and numerical (PIC) spectra of density fluctuations at different times.

whether observed features of the modulational instability driven by the strong pumping wave correspond to theoretical insights into this phenomenon. The comparison of unstable spectra observed in PIC simulations with their theoretical predictions is presented in Fig. 3 and demonstrates good agreement. In order to check that the observed instability does not depend on numerical effects, we carried out a simulation experiment with the doubled size of the grid cell and observed the same evolution of the unstable spectrum.

III.C Steady-state turbulence

Before we simulate long-time plasma heating, let us explore whether the hybrid and PIC simulations agree with each other at the stage of the steady-state turbulence. Although the hybrid model contains free dissipation parameters such as γ_l and γ_s, the pump power appears to depend weakly on their values within a rather wide range. For the parameters $\gamma_l = 5 \cdot 10^{-5}$, $\gamma_s = 5 \cdot 10^{-5}$ and $\kappa = 0.05$, agreement between simulations is shown in Fig. 4. It is seen that beam energy losses (Fig. 4a) as well as the energy accumulated in plasma waves (Fig. 4b) can be adequately described by the hybrid model.

Strong dissipation produced by the modulational instability results in the situation when the energy

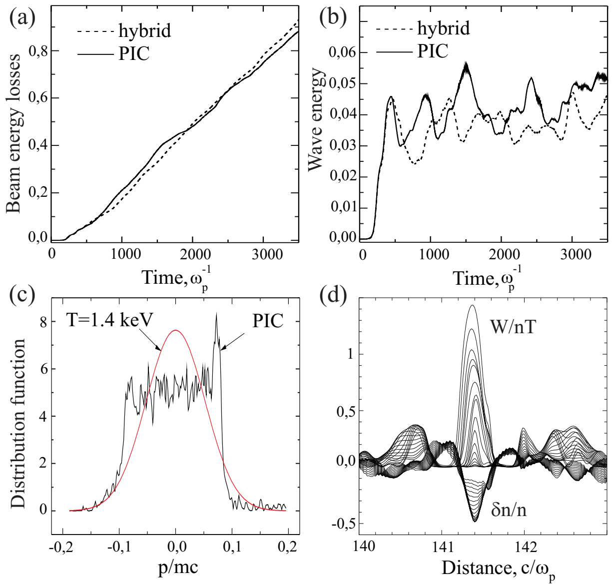

Figure 4: Comparison of PIC and hybrid simulations at the stage of the steady-state turbulence: (a) beam energy losses, (b) energy accumulated in plasma waves, (c) momentum distribution of plasma electrons observed in PIC simulations at the point of maximum heating at the moment $\omega_p t = 2000$ and maxwellian distribution corresponding to the temperature predicted by the hybrid model. (d) Dynamics of the localized wave packet and corresponding density well during the forced collapse ($\omega_p t = 2069 \div 2269$).

of Langmuir oscillations is almost completely transformed to the thermal energy of plasma electrons. The electron distribution function in this case is marked by the plateau which is formed within the momentum range corresponding to the local quiver velocity (Fig. 4c). With the increase in thermal energy turbulence goes to the regime $W < nT$, which is governed by the same nonlinear processes as were found in the clamp-driven case. The rate of nonlinear dissipation of resonant waves is determined by their scattering off long-wavelength density fluctuations $\nu = \sqrt{\langle \delta n^2/n^2 \rangle} \sim W/nT$, whereas the further energy transfer through the turbulent spectrum is produced by either scattering or the forced collapse. The typical collapse event observed in hybrid simulations is shown in Fig. 4d. It is seen that the localized wave packet narrows and burnouts as the density well deepens under the ponderomotive force.

With the increase in temperature beam-excited turbulence tends to operate in the regime $\nu < \Gamma$, when scattering off density fluctuations is not able to destroy nonlinear correlations associated with beam trapping. Thus, under weak nonlinear dissipation imposed by turbulence, the beam absorbs some energy of resonant waves by itself and saturates the pumping power. As to the turbulence, it plays an auxiliary role in this regime just delivering a given energy flux to the dissipation region. As seen in Fig. 5a, where simulations with various beam density are presented, the pumping power $\tilde{P}$, integrated over the length $L = 600\, c/\omega_p$, is indeed found to be independent on time-varying turbulence parameters. The power saturation in the turbulent steady state in this regime is level with the maximum power lost by the beam at the dynamic stage (Fig. 5b), when trapping of beam particles by the field of the regular resonant wave is the only nonlinear effect. Thus, the regime of the constant pump power is

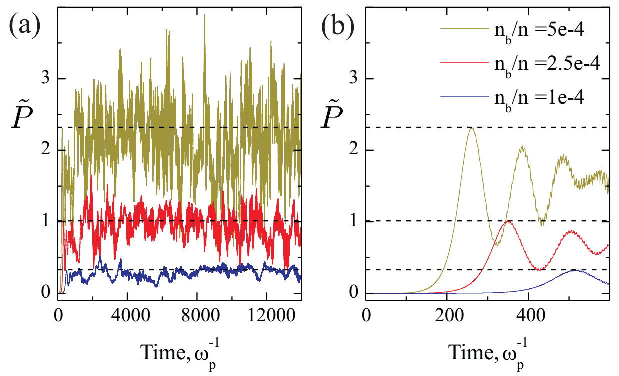

Figure 5: (a) Evolution of power lost by the beam in the plasma of the length $L = 600c/\omega_p$ for various beam densities and the fixed angular spread $\Delta\theta = 0.2$; (b) the same power at the dynamic stage (fragment of (a)).

established as natural consequence of nonlinear evolution of beam-driven turbulence during long-time beam injection. The level of power saturation in this regime does not depend on turbulence nature and is determined by nonlinear interaction of beam particles with resonant oscillations only.

ACKNOWLEDGMENTS

We thank K.V.Lotov for useful discussions. The work is supported by Russian Ministry of Education grant RNP2.1.1/3983.

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