TAIL-WAVING SYSTEM FOR ACTIVE FEEDBACK STABILIZATION OF FLUTE MODES IN OPEN TRAPS

A. D. Beklemishev

Budker Institute of Nuclear Physics and Novosibirsk State University,
beklemishev@inp.nsk.su
Feedback Stabilization

The general idea of feedback stabilization is to
  • detect and diagnose the plasma perturbations,
  • analyze the diagnostic signal and generate the necessary response,
  • reduce the perturbation using the control system and the response signal.

“... electronic scheme is created outside of the plasma, that interacts with the plasma via oscillation fields. Parameters of the scheme are chosen in such a way that the whole system, consisting of the plasma and the electronic scheme, is stable.”

---

Plasma control systems

- **Electrostatic**, successfully used on OGRA-III
  
  [1] V.V. ARSENIN, V.A. CHUYANOV *Sov. Phys. Uspekhi*, 20, 736 (1977);

- **Axial potential control (via line-tying),**
  
  

- **Ponderomotive,**
  

- **Magnetic:** Two instances cited in [1], and
  

  (Resistive wall-mode stabilization.)

The magnetic control systems have the advantage of not relying on high voltage in vacuum and plasma-electrode contacts. Unfortunately, existing proposals are not really feasible.
Magnetic control schemes

1) Feedback via Ioffe-rods; 2) Control via axial field (short traps) \(^2\).

Our aim is to devise a new control scheme for long axially symmetric traps (with \( l \propto \xi^1 \)), using perturbations of the transverse field.

\(^2\)Figure from Arsenin & Chuyanov, Sov.Phys.Uspekhi, 1977
Influence of local transverse fields

Local application of a transverse magnetic field (light-blue) can cause $z$-dependent displacement, $m = 1$, or a shape change, $m > 1$, of the plasma column. It does not move flux tubes as a whole directly since the influence is symmetric. But such motion may arise due to $z$-dependence of plasma parameters. Hence,

- the influence of fields is *indirect*,
- *in symmetric traps the feedback fields should also be symmetric* to produce maximum effect.
The magnetic system consists of two symmetrically placed belts, each containing a number of coils for production of multipole transverse fields.
Operation sequence - 1

Unperturbed fluxtube
For flute modes, $\phi(\ell) = \text{const}$, the transverse plasma displacement is inversely proportional to the magnetic field.
The “tail swing” in the direction of the initial flute displacement also retracts the central part. Displacement becomes anti-ballooning.
The flute starts moving back. The feedback field is reduced proportionally.
Operation sequence - 5

Flute mode $m=1$ w/feedback decay
Operation sequence - 6
Reduced-MHD model

Starting from the quasi-neutrality $\text{div} j_\parallel b + \text{div} j_\perp = 0$, and obtaining $j_\perp$ from the momentum equation, we get

$$
(\mathbf{B}\nabla) \frac{j_\parallel}{B} + c\nabla p \cdot \nabla \times \frac{\mathbf{b}}{B} - c \frac{\mathbf{b}}{B} \cdot \nabla \times \rho \frac{d\mathbf{v}}{dt} = 0. \tag{1}
$$

Then introduce $\Phi = c \int \varphi dt$ such that the plasma displacement is

$$
\xi = \frac{\mathbf{b}}{B} \times \nabla \Phi, \tag{2}
$$

and take (in the reduced-MHD way) $\tilde{p} \approx -\xi \nabla p_0$ and $\tilde{j}_\parallel = -\frac{c}{4\pi} \Delta_\perp A_\parallel$. 

Reduced-MHD model

The quasi-neutrality condition becomes

\[(B \nabla) \frac{1}{B} \Delta_\perp A_\parallel + \nabla_\perp c_\text{A}^{-2} \nabla_\perp \ddot{\Phi} + K \nabla_y^2 \Phi = 0,\] (3)

where \(K = 8\pi p'_0 \kappa / B^2\) is the pressure-weighted normalized field curvature, \(\kappa = e_r (b \nabla) b\); \(\nabla_y\) represents derivative in the direction of the binormal to the magnetic field, and \(c_\text{A}\) is the Alfvén speed.

The closure condition for equation (3) is the parallel component of the Ohm’s law,

\[A_\parallel = - (b \nabla) \Phi.\] (4)
Reduced-MHD model

We can introduce the externally induced (by the feedback coils) small transverse magnetic field, \( A_{\parallel f} \), such that \( \Delta_{\perp} A_{\parallel f} = 0 \). In accordance with the Ohm’s law, it is accompanied by displacement:

\[
\Phi_f = - \int A_{\parallel f} d\ell. \tag{5}
\]

The total perturbed field is a sum of the mode field and the feedback field, then

\[
(B \nabla) \frac{1}{B} \Delta_{\perp} \frac{1}{B} (B \nabla) \Phi - \nabla_{\perp} c_A^{-2} \nabla_{\perp} \ddot{\Phi} + K \nabla_{\perp}^2 \Phi =
\]

\[
= \nabla_{\perp} c_A^{-2} \nabla_{\perp} \ddot{\Phi}_f - K \nabla_{\perp}^2 \Phi_f. \tag{6}
\]

The left-hand side describes Alfvén waves, interchange and ballooning modes, as its terms describe the field-line tension, plasma inertia, and the curvature drive.
Integrate Eq. (6) along a field line and assume electrostatic approximation, $\Phi = \text{const}$, on each field line. Then the highest-order term is expressed via current densities into the end-plates:

$$\langle K \nabla_y^2 \rangle \Phi - \langle \nabla_\perp c_A^{-2} \nabla_\perp \rangle \ddot{\Phi} = \frac{4\pi}{cL} \left( \frac{j_{\parallel a}}{B_a} - \frac{j_{\parallel b}}{B_b} \right) +$$

$$+ \langle \nabla_\perp c_A^{-2} \nabla_\perp \dot{\Phi}_f \rangle - \langle K \nabla_y^2 \Phi_f \rangle.$$ (7)

Here the angular brackets represent weighted field-line averages as $\langle f \rangle = L^{-1} \int (f / B) d\ell$, while indices $a$ and $b$ stand for values at opposite end-plates.
Reduced-MHD model

- By choosing a suitable amplitude and sign of $\Phi_f \propto \Phi$ it is possible to compensate or overcompensate the curvature drive.
- In general the inertial and the curvature source-terms are comparable to each other and to the left-hand side terms. Their relative value can be adjusted by particular placements of control coils, which coincide with nodes of $\Phi_f$.
- Due to quite different parameter scalings of source terms it is highly unlikely that they will vanish together.
Possible coil placements

- The “anti-ballooning placement” of control coils is to place them near inflection points of the field lines, so that $\Phi_f$ would change sign together with the curvature, $K$, thus maximizing the integral $\langle K \nabla_y^2 \Phi_f \rangle$. Placing $\Phi_f$ nodes in such a way as to eliminate inertia, even without stabilizers we are still left with a small but sign-definite curvature source. However, in this case $\Phi_f/\Phi > 1$.

- The “inertial placement” is to minimize the curvature source term, so that the plasma response will be proportional to the second time derivative of the control current.
Estimate of needed power

Assume that each belt of the feedback system is $\ell_f = 50\,cm$ long, has a radius of $R = 20\,cm$, can create radial displacements of $\xi = 1\,cm$ in the midplane, and has a frequency of $\nu = 10\,kHz$.

To create displacement of the plasma by $\xi$, the field produced by the coils should be $B_f = B_0 \xi / \ell_f$, where $B_0$ is the field in the midplane. This field is created in the volume of $\sim \pi R^2 \ell$ and is renewed with the frequency $\nu$. If $B_0 = 10^4\,gauss$, then the needed feedback field is $B_f \approx 200\,gauss$, and without recuperation

$$W = 2 \cdot \nu \cdot \frac{B_0^2}{8\pi} \cdot \pi R^2 \xi^2 / \ell_f \approx 200\,kW.$$  

Note that $\nu \sim L^{-1}$ so that the power consumption scales as

$$W \propto \xi^2 L^{-1} \ell_f^{-1}!$$
Summary & discussion

Purely electromagnetic plasma-control system that is independent of line-tying or plasma conductivity to the end-plates is proposed.

- The tail-swinging control system should be symmetric in $z$ with placement of coil belts around bad-curvature regions, in order to impose the “anti-ballooning” modulation on flute modes;

- Since the plasma reacts to feedback with Alfvén retardation, at $\beta \sim 1$, when $\gamma \sim L/c_A$, the localized coil system will not work. Stabilization of ballooning instability requires a distributed system of coils;

- FLR effects are essential, as the control system can only influence a few azimuthal modes with only a single radial harmonic each;

- The system should be able to create displacements of order of the diagnostic sensitivity or the fluctuation level. The required power for GDT parameters seems reasonable.
I hope that the proposed magnetic control system will find its way to experimental testing.
“Sausage the Rope-Crawler”

If Sausage stays straight, it is going to fall.

Gravity models bad curvature, while balloons correspond to good-curvature regions.
“Sausage the Rope-Crawler”

Flexibility allows Sausage to create leverage for balloons.

Note that inertia causes simultaneous counter-swing in the middle. It could be useful, if the middle goes over the top!