Chaotic Neoclassical Transport, Damping and Wave Couplings from θ-Ruffled Separatrices

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Locally Trapped Particles, Separatrices

Even weak localized trapping of particles produces strong transport and damping effects in low-collisionality plasmas. Hundreds of neoclassical theory papers since 1960’s; but very few experimental tests.

Prior theory analyzed collisional scattering on \( \theta \)-symmetric separatrices only, obtaining \( \nu^{1/2} B^{-1/2} \) scalings.

Pure electron plasma experiments characterize transport and wave damping from a well-controlled trapping barrier. Adding \( \theta \)-dependent separatrix “ruffles” or temporal variations gives enhanced effects with \( \nu^0 B^{-1} \) scalings.

Recent theory analyzes this as “chaotic” (rather than “collisional”) separatrix dissipation.
Pure Electron plasmas: Excellent confinement, $\tau \sim 100$ sec

+ Controlled Separatrix + $\theta$-Ruffle
+ Controlled Tilt "Global Asymmetry"

central density: $n_0 \approx 1.5 \times 10^7$ cm$^{-3}$
central potential: $\varphi_0 \approx -30$ V
plasma radius: $R_p \approx 1.2$ cm; $L_p = 50.$ cm
equilibrium temperature: $T \approx 1$ eV
magnetic field: $1 < B \leq 20$ kG
$\mathbf{E}\times\mathbf{B}$ rotation frequency: $f_E(B) \approx 200$ kHz $(B/1\text{kG})^{-1}$
avxial bounce frequency: $f_b(T) \approx 600$ kHz
$e\text{--}e$ collision frequency: $\nu_{ee}(n,T) \approx 140$ sec$^{-1}$
neutral pressure: $P \approx 10^{-11}$ Torr
enables & DAMPS TPDM

**NO external asymmetries such as magnetic tilt**

**separatrix + \cos(m\theta) ruffle**

**DAMPS**

- \(m_\theta, k_z\) plasma modes
- \(m_\theta = 0, k_z\) plasma modes
- bulk plasma EXPANSION & loss
- \(m_\theta, k_z = 0\) diocotron modes

**WITH external asymmetries**

\[ \nu_{\langle r^2 \rangle} \equiv \frac{1}{\langle r^2 \rangle} \frac{\partial \langle r^2 \rangle}{\partial t} \]

traditional theory

\[ \nu_{\langle r^2 \rangle} \propto \left( \frac{\nu_{\text{coll}} B}{1} \right)^{-1/2} \]

chaotic theory

\[ \nu_{\langle r^2 \rangle} \propto \left( \frac{\nu_{\text{coll}} B}{1} \right)^{-1} \]
squeezing sectors

$V_{sq} + \Delta V_m \cos(m\theta)$ or $\delta B/B$

$\phi_s(r, \theta) = \phi_{s0}(r) + \Delta \phi_m(r) \cos[m(\theta - \theta_m)]$

$\theta_B = \tan^{-1}(B_y / B_x)$

"Global Asymmetry" from Magnetic Tilt: $\phi_a(r, z, \theta) \propto \varepsilon_B \cos(\ell \theta - \theta_B) \frac{z}{L_p}$ where $\ell = 1$
Differing Drifts cause Separatrix Discontinuity, Smoothed by Collisions

Rosenbluth, Ross, & Kostomarov – 1972
Hilsabeck & O'Neil – 2003
\( \theta \)-Symmetric separatrix

\[
\varphi_a[R_w, \theta, z] = \varepsilon_a 2eN_L \frac{z}{R_w} \cos[\theta - \theta_B]
\]

\[ n_{a[\theta_a \pm \frac{\pi}{2}]}^{tr} \propto \varphi_a \sqrt{\frac{\nu_{ee}}{\omega_E}} \propto \varphi_a B^{0.5} \]

and \( j_r = \left( \frac{c}{rB} \right) n_a \frac{\partial \varphi_a}{\partial r} \)

gives \( j_r \propto \varphi_a^2 B^{-0.5} \)
θ-ruffled (cos2θ) separatrix

Prior theory presumed \( \Delta \theta = 0 \)

Experiments rotate \( \theta_B \) and observe \( \sin^2(\theta_B - \theta_2) \) “petals”

\[
n_{a_0}^{tr} \approx \frac{\varphi_2 \varphi_a \cos^2(\Delta \theta) B^0}{(c/rB_n a \partial \varphi_a / \partial r} \]

and \( j_r = (c/rB) n_a \partial \varphi_a / \partial r \) gives

\[
j_r \propto \varphi_a \varphi_a^2 \sin^2[\Delta \theta] B^{-1}
\]
m=2 ruffle-induced $\mathcal{V}_{r^2}$ shows distinctive $\sin^2(\theta_B - \theta_m)$ signature.
Chaotic transport is proportional to applied ruffle $\Delta V_2$

$\nu_{q\langle r^2 \rangle} = \frac{\pi}{2}$

$\nu_{q_{B}} = \frac{\pi}{2}$

$\nu_{B} = 1 \text{ kG}$

$\nu_{B} = 10 \text{ kG}$

$\nu_{\varepsilon} = 4 \text{ mRad}$

$\nu_{\varepsilon} = 7 \text{ mRad}$
Collisional and m-Ruffled chaotic transport coefficients

\[ \mathcal{V}_{\langle r^2 \rangle} = C_{cA} \hat{\varepsilon}_B^2 + C_{mA} \hat{\varepsilon}_B^2 \Delta \hat{V}_m \sin^2(\alpha) + C_{cK1} \hat{\varepsilon}_B^2 + C_{cK2} \Delta \hat{V}_m^2 + \mathcal{V}_{\langle r^2 \rangle}^{(bkg)} \]

\[ \Delta \hat{V}_m = \Delta V_m / \text{Volt} \]
\[ \hat{\varepsilon}_B \equiv \varepsilon_B / \text{mRad} \]
\[ \Delta W_c / \Delta \phi_2 \]

\[ \phi_{s0} = 0.5T, \Delta \phi_2 = 0.1T, \]

\[ R = 30, \alpha = 1 \]

Chaotic Enhancement
Gratuitous $\delta B/B \sim 10^{-3}$ mirror

-- gives strong transport in "original placement"

-- up to 5x lower in "new placement"

Stronger mirrors give the same $\nu_{\langle r^2 \rangle}$
Temporal separatrix variations $\Delta \phi_t$ also cause strong chaotic transport

$\Delta \phi_t \sim 0.2 \text{ T}$
$\sim 3 \ W_{\text{collis}}$

$\Delta \nu_{<r^2>} \propto \Delta \phi_t$

Effective when $f_{\text{noise}} > f_{\text{ExB}}$

$V_{sq} = -6V + V_{\text{noise}}$
$\varepsilon_B = 0.001$
$B = 12 \text{kG}$

$\nu_{<r^2>} \approx 0.06/\text{sec}$
$\nu_{<r^2>} \approx 0.02/\text{sec}$

$V_{\text{noise}} (\geq 20\text{kHz}) = 0.2V$
enables & DAMPS TPDM

\( \gamma_{TPDM} \)

NO external asymmetries such as magnetic tilt

DAMPS
\( m_\theta, k_z \)
plasma modes

\( \gamma_{TPDM} \)

adds nonlinear wave - wave COUPLINGS

separatrix + \( \cos(m\theta) \) ruffle

DAMPS
\( m_\theta=0, k_z=0 \)
diocotron modes

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bulk plasma EXPANSION & loss

WITH external asymmetries
Trapped-Particle Diocotron (Drift) Mode
(\ell=1, \text{ z-anti-symmetric})

Launch Wave

Observe Damping

\[ e^{-\gamma t} \]
TPDM Damping:
Easily measured

Easily *increased* with artificially-induced scattering

*For large B, $\gamma \propto B^{-0.5}$
Magnitude consistent with $\theta$-symmetric collisional theory

*For small B, $\gamma \propto B^{-1}$
Due to “chaotic” dynamics on gratuitous separatrix ruffles
Chaotic: damping $\gamma \ast B \propto B^0$
Collisional: $\gamma \ast B \propto B^{1/2}$

for both
Collisional
Chaotic static ruffle
Chaotic wave-ruffled

\[ \gamma \propto B^{-1/2} \]

\[ \gamma \propto B^{-1} \]

\[ \Delta V_2 = 2V \]
\[ Q = 0.02 \]

\[ \gamma^{(V_2)} \approx 3370V_2B^{-1} \text{ / sec} \]

\[ \gamma^{(Q)} \approx 1.8 \times 10^5 QB^{-1} \text{ / sec} \]
enables & DAMPS TPDM

DAMPS $m_\theta, k_z$
plasma modes

$\gamma_{TPDM}$

NO external asymmetries
such as magnetic tilt

adds nonlinear wave - wave COUPLINGS

separatrix + $\cos(m\theta)$ ruffle

DAMPS $m_\theta=0, k_z$
plasma modes

WITH external asymmetries

$\nu_P$

bulk plasma EXPANSION & loss

DAMPS $m_\theta, k_z=0$
diocotron modes

Wave-wave couplings
Resonant Parametric Decay:

$m=2$ pump wave decays into two $m=1$ daughter waves.

Collisional dissipation alone gave wrong damping, wrong phase shifts (dashed curves).
Chaotic dissipation from wave-induced ruffle gives new nonlinear coupling new terms

\[
\frac{dA_2}{dt} = -K_0 A_1^2
\]

\[
\frac{dA_1}{dt} = (K_0 + iK_1) A_2 A_1^* + \delta\omega A_1
\]

\[-(K_2 + iK_3)|A_2| A_1 - \gamma_1 A_1\]

With ruffle-induced dissipation, quantitative description of late-time evolution is obtained
enables & DAMPS TPDM

DAMPS $m_\theta, k_z$ plasma modes

add non-linear wave-wave COUPLINGS

NO external asymmetries such as magnetic tilt

separatrix + \cos(m \theta)\text{ ruffle}

DAMPS $m_\theta=0, k_z=0$ diocotron modes

bulk plasma EXPANSION & loss

WITH external asymmetries

\gamma_{md}
Asymmetry-Induced Diocotron Mode Damping:
Tilt + Diocotron Mode $\Rightarrow$ Sloshing Currents across separatrix;
Collisions or Chaos dissipate these Currents, as for $\mathcal{V}_{\langle r^2 \rangle}$

Indeed, the *same* quadrupole petals
are observed in diocotron mode damping $\gamma_{2d}(\theta_B)$
as as are observed in bulk transport $\mathcal{V}_{\langle r^2 \rangle}(\theta_B)$
enables $\text{DAMPS TPDM}$

$\text{DAMPS } \begin{pmatrix} m_\theta, k_z \end{pmatrix}$

$\text{plasma modes}$

$\text{DAMPS } \begin{pmatrix} m_\theta = 0, k_z = 0 \end{pmatrix}$

$\text{diocotron modes}$

$\gamma_{mk}$

$\text{NO external asymmetries such as magnetic tilt}$

$\text{adds nonlinear wave-wave COUPLINGS}$

$\gamma_{mk}$

$\text{separatrix + cos(m\theta) ruffle}$

$\text{DAMPS}$

$\begin{pmatrix} m_\theta = 0, k_z = 0 \end{pmatrix}$

$\text{diocotron modes}$

$\text{WITH external asymmetries}$

$\text{bulk plasma EXPANSION & loss}$
$\gamma_{11} : m=1, \ k_z = \frac{1\pi}{L_p}$ Lamgmuir Mode Damping

$\gamma_{11}^{(M)}$ from $\delta B/B \sim 10^{-3}$ mirror

$\gamma_{11}^{(V_{sq})}$ from $V_{sq}(t)$

(Ruffles not yet investigated)
Summary

Even weak localized trapping of particles ($\delta B/B \sim 10^{-3}$) produces strong "superbanana" transport and damping in low-collisionality plasmas.

**Collisional** separatrix scatterings on

$\theta$-symmetric separatrices give $\nu^{1/2} B^{-1/2}$ scalings.

Adding $\theta$-dependent separatrix “ruffles” or temporal variations gives enhanced effects with $\nu^0 B^{-1}$ scalings.

Recent theory analyzes this as “chaotic” (rather than “collisional”) separatrix dissipation.
NNP.ucsd.edu: Trapped-Particle Mediated Modes, Damping and Transport

Resonant Chaotic transport and Damping from θ-ruffled Separatrices

D.H.E. Dubin, C.F. Driscoll and Yu.A. Tsidulko
Neoclassical transport caused by collisionless scattering across an asymmetric separatrix

A.A. Kabantsev, T.M. O'Neil, Yu.A. Tsidulko and C.F. Driscoll
Resonant Drift-Wave Coupling Modified by Nonlinear Separatrix Dissipation

A.A. Kabantsev and C.F. Driscoll
Trapped-Particle-Mediated Collisional Damping of Non-Axisymmetric Plasma Waves

T.J. Hilsabeck, A.A. Kabantsev, C.F. Driscoll, and T.M. O'Neil

A.A. Kabantsev and C.F. Driscoll

A.A. Kabantsev and C.F. Driscoll
Trapped-Particle Modes and Asymmetry-Induced Transport in Single-Species Plasmas

A.A. Kabantsev, C.F. Driscoll, T.J. Hilsabeck, T.M. O'Neil, and J.H. Yu