Chaotic Neoclassical Transport at Azimuthally Perturbed Or Wave-Mingled Separatrix

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Non-neutral plasmas are confined by static electric and magnetic fields in a Penning-Malmberg trap.

Cylindrical symmetry, single species $\Rightarrow$ long confinement time

\[ P_\theta = \sum_{i=1}^{N} \left[ m \sqrt{\theta_i r_i} + \frac{eB}{2c} r_i^2 \right] = const. \]
Neoclassical transport is the dominant loss process in non-neutral plasmas

- But 20+ years of experiments have not made contact with the standard neoclassical theory (banana/plateau/Pfisrch-Schluter).
- Instead, experiments are consistent with a new theory for particles trapped in local field ripples – analogous to neoclassical ripple (superbanana) transport in stellarators,
- **but in previously undiscovered “chaotic” regime caused by asymmetric (“ruffled”) separatrix.**
- This talk will present main results of the new transport mechanism.

\[
\frac{\Delta \phi_s(\theta,t)}{T} \sim 0.1
\]

\[\varepsilon_B \equiv \frac{B_\perp}{B_z} \sim 0.001\]

**typical field error scales:**
Schematics of the experiments and plasma parameters

\[ V_{sq} + \Delta V_m \cos(m\theta) \]
or \[ \delta B/B \]

\[ V_a(z) \]

\[ V_a(R_w, \theta, z) = \varepsilon_B z(2eN_L / R_w) \cos(\theta - \theta_B) \]

\[ \varepsilon_B \equiv B_\perp / B_z \leq 10^{-3} \]
\[ \theta_B \equiv \tan^{-1}(B_y/B_x) \]

- \[ n_0 \approx 1.6 \times 10^7 \text{ cm}^{-3} \]
- \[ R_p \approx 1.2 \text{ cm} \]
- \[ R_w = 3.5 \text{ cm} \]
- \[ L_p = 49 \text{ cm} \]
- \[ \phi_0 \approx -28 \text{ V} \]
- \[ T \leq 1 \text{ eV} \ (\lambda_D \approx 0.1R_p) \]
- \[ 0.4 \leq B \leq 20 \text{ kG} \]
- \[ f_E \approx 230 \text{ kHz} / \text{B(kG)} \]
- \[ f_b(T) \approx 430 \text{ kHz} \]
- \[ \nu_{ee} \approx 140 \text{ sec}^{-1} \ll f_E \leq f_b \]
- \[ P \approx 10^{-11} \text{ Torr} \]
for axial bounce frequencies \( f_b \gg f_E \),

**trapped** drift orbits displaced by \( \Delta r \sim \varepsilon_B L_p \)

**passing** drift orbits unaffected by tilt \( \varepsilon_B \equiv B_\perp/B_z \)

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standard neoclassical for \( \nu < f_E \)

\[
D_r \sim f_E \Delta r^2 F_M(\phi_{s0}) W_c \propto \sqrt{\nu/B}
\]

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collisional spread for a drift-rotation period

\[
\Delta W_c \equiv T(\nu/2\pi f_E)^{1/2} (\phi_{s0}/T)^{1/2} \approx 0.025\text{eV} \; B_{kG}^{1/2}
\]

"chaotic" (de)trapping becomes important if

\[
\Delta \phi_m \geq \Delta W_c , \text{ or when } \Delta \phi_t \geq \Delta W_c
\]

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Poloidal projection of a typical superbanana orbit: h- **helically trapped**, t- **toroidally trapped** portions

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Phase angle $\alpha$ between separatrix and asymmetry potential must be nonzero to see collisionless diffusion

$$\phi_s(r, \theta) = \phi_{s0}(r) + \Delta \phi_m(r) \cos[m(\theta - \theta_m)]$$

$$\delta \phi_a(r, \theta, z) \propto \varepsilon_B z \cos[l(\theta - \theta_B)]$$

$$\alpha \equiv \theta_B - \theta_m$$

Particle orbits are trapped and detrapped at same radius -- no radial steps

Particle orbits are trapped and detrapped at different radii -- radial steps

$$D_r \propto \Delta \phi_m \varepsilon_B^2 \sin^2 \alpha$$
NEOCLASSICAL RADIAL DIFFUSION COEFFICIENT

\[
D_r(r) = \bar{f}_E \left[ \frac{(\delta \phi_L - \delta \phi_R)}{\partial \Phi_e/\partial r} \right]^2 \frac{1}{4} F_M(\phi_{s0}) \left\{ \frac{\Delta W_c D_{cA} + \Delta \phi_2 D_{2A} \sin^2 \alpha}{D_*} \right\}
\]

Scaled diffusion coefficients versus the relative perturbation strength

\[
\frac{D_*}{\Delta W_c} = D_{cA} + \frac{\Delta \phi_2}{\Delta W_c} D_{2A}
\]

\[
\Delta \phi_2 / \Delta W_c
\]

\[
\sin^2 \alpha = 1
\]

\[
D_* \approx 4(\Delta \phi_2 + 0.88\Delta W_c e^{\frac{1}{0.88 \Delta W_c}})
\]

convenient approximation

\[
\Delta \phi_2 / \Delta W_c
\]
Measurements of neoclassical transport rate

\[ \nu_{\langle r^2 \rangle} = 0.0045 + 0.033 \varepsilon_B^2 \]

**Graph:**
- \( B = 6 \text{kG} \)
- \( V_{sq} = 6 \text{V} \)
- \( \Delta V_2 = 0 \text{V} \)

**Equations:**

\[ D_* \approx 4 \times (\Delta \phi_2 + 0.88 \Delta W_c e^{-\frac{\Delta \phi_2}{0.88 \Delta W_c}}) \]

\[ \nu_{\langle r^2 \rangle} \approx 0.11 \times (\Delta V_2 + 0.66 e^{-\frac{\Delta V_2}{0.66 V}}) \]

\[ \Rightarrow \left\langle \frac{\Delta \phi_2}{\Delta W_c} \right\rangle_r \approx \frac{4 \Delta V_2}{3 \text{V}} \]

**Graph:**
- \( \varepsilon_B = 1.45 \text{mrad} \)
- \( B = 6 \text{kG} \)
- \( V_{sq} = 6 \text{V} \)

\[ \sin^2 \alpha = 1 \]
Enhancement (and suppression) of neoclassical transport by a ruffled separatrix

\[
v_{r^2} = C_{CA}(\Delta V_m)\varepsilon_B^2 + C_{mA}\varepsilon_B^2\Delta V_m \sin^2 \alpha + C_{cK1}\varepsilon_B^2 + C_{cK2}\Delta V_m^2 + v^{(bkg)}_{r^2}
\]
\[
\alpha \equiv \theta_B - \theta_m
\]
Measured expansion rate at fixed $\Delta V_2$

$$\nu_{<r^2>} = C_{cA}(\Delta V_m)\varepsilon_B^2 + C_{mA}\varepsilon_B^2\Delta V_m \sin^2 \alpha$$

$$+ C_{cK1}\varepsilon_B^2 + C_{cK2}\Delta V_m^2 + \nu^{(bkg)}_{<r^2>}$$

$\Delta V_2 = 1.1V$

$V_{sq} = 6V$

$B = 6kG$

$C_{2A} \approx 0.056$

$C_{cA}(0V) \approx 0.033$

$C_{cA}(1.1V) \approx 0.019$
Measured expansion rate at fixed $\varepsilon_B$

\[ \nu_{<r^2>} = C_{cA} (\Delta V_m) \varepsilon_B^2 + C_{mA} \varepsilon_B^2 \Delta V_m \sin^2 \alpha \]

\[ + C_{cK1} \varepsilon_B^2 + C_{cK2} \Delta V_m^2 + \nu_{<r^2>}^{(bkg)} \]

\[ \Delta V_2 = 1.1V \]

\[ B = 4kG \]

\[ V_{sq} = 6V \]

\[ \theta_B [\pi] \]

\[ C_{2A} \approx 0.080 \]

\[ C_{cK2} \approx 0.025 \]
Magnetic scaling of various transport terms (solid lines – theory)

\[ \nu_{<r^2>} = C_{cA} \varepsilon_B^2 + C_{mA} \varepsilon_B^2 \Delta V_m \sin^2 \alpha \]
\[ + C_{cK1} \varepsilon_B^2 + C_{cK2} \Delta V_m^2 + \nu^{(bkg)}_{<r^2>} \]

- \( C_{cA} \approx 0.10B^{-0.62} \)
- \( C_{2A} \approx 0.35B^{-1} \)
- \( C_{cK1} \approx 0.51B^{-2.7} \)
CONCLUSIONS

• All magnetic confinement devices have some level of magnetic or electric ‘ripples’ that provide separatrices and locally trap particles
• These separatrices are never perfectly symmetric ($\Delta \phi_s(\theta,t) \neq 0$) or perfectly aligned ($\varepsilon_B \neq 0, \alpha \neq 0$) with other asymmetries
• Chaotic flavor of neoclassical transport presented here could be important in these devices at low collisionality:

$$D_r \propto \nu^0 B^{-1} \Delta \phi_s(\theta,t) \varepsilon_B^2 \sin^2 \alpha$$

• This loss mechanism is the dominant bulk transport process in our non-neutral plasma experiments at high rigidity and low collisionality, when the separatrix layer collisional width $\Delta W_c$ falls below the typical ruffle scale $\Delta \phi_s(\theta,t)$