The flute mode fluctuations and associated radial transport in the GAMMA10 divertor
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The GAMMA10 tandem mirror is planning to replace one of anchor mirror cell by an axi-symmetric divertor mirror.
The main purpose of the divertor is to realize the effective evacuation of core plasma into a dipole region outside the divertor mirror. The plasma in the core region is stabilized by creating high pressure plasma in the remaining non-axisymmetric anchor mirror cell and the plasma in the peripheral region is stabilized by the divertor mirror. The large classical transport around magnetic null easily makes plasma pressure radial profile unstable to flute modes. And it can enhance the radial transport in the GAMMA10 A-divertor.
The purpose of this presentation is to make clear the flute mode stability and its effects to the radial transport in the system with mixed stability mechanism of min.B and plasma compressibility.

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The GAMMA10 A-divertor  *(is not a final design)*

In the GAMMA10 A-divertor one anchor mirror cell is replaced by an axisymmetric divertor mirror cell. The GAMMA10 A-divertor uses the vacuum chamber and coils of the present GAMMA as much as possible. The role of magnetic null at x-point, which makes a round of z-axis.

- Unmagnetized electrons can move freely along the magnetic null, which has an effect of short circuit of electrostatic perturbation.
- Thus electrostatic perturbation is stabilized on the separatrix magnetic surface.
- Conservation of ion magnetic moment $\mu$ is broken near x-point. So ions coming from the central cell can escape to the dipole region.
- Thus the magnetic null has a role of punching a hole on the magnetic flux tube. Almost all the ions, diffused in the central core region, move along a magnetic field line to the x-point, and escape to the dipole region.
Flute interchange modes are stabilized by a high pressure creating in anchor mirror cell in the core region. Flute interchange modes in the peripheral region are stabilized by a plasma compressibility in divertor mirror.

\[ \int \left[ \hat{p}_\parallel (B) + \hat{p}_\perp (B) \right] \kappa_\psi \frac{d\ell}{B} \geq 0 \]

which is traditionally used to estimate the non-axisymmetric mirror stability

\[ \kappa = \kappa_\psi \nabla \psi + \kappa_\theta \nabla \theta , \quad p_{\parallel,\perp} (\psi, B) = \hat{p}_{\parallel,\perp} (B) \nu (\psi) \]

is represented by a separation of variable

High plasma pressure is required in a remaining anchor cell for flute mode stability.

\[ \frac{\partial U}{\partial \psi} \frac{\partial p U^\gamma}{\partial \psi} \geq 0 \]

which is applied to axi-symmetric divertor mirror stability

\[ U = \int \frac{d\ell}{B} \]

is a magnetic field line specific volume.

We change the non-axisymmetric stability criterion to the axisymmetric one so as to use axisymmetric equations. So we include the effect of the axial pressure profile into \( U \) to apply the axisymmetric equations to GAMMA10.
Specific Volume $U$

The stability criterion \( \int \frac{[\hat{p}_\parallel + \hat{p}_\perp] \kappa_\psi}{B} d\ell \geq 0 \) in the core region can be written as

\[
\frac{\partial U}{\partial \psi} < 0 \quad \text{if } U \text{ is defined as } U \equiv \int \frac{[\hat{p}_\parallel + \hat{p}_\perp]}{B} d\ell
\]

\[
\Rightarrow \quad \frac{\partial U}{\partial \psi} = -2 \int \frac{[\hat{p}_\parallel + \hat{p}_\perp] \kappa_\psi}{B} d\ell
\]

In the steady state, therefore, plasma is stable in terms of new $U$, if

- in the core region (stabilized by min.B)
  \[
  \frac{\partial p}{\partial \psi} < 0 \text{ is satisfied because } \frac{\partial U}{\partial \psi} < 0.
  \]
- in an axisymmetric divertor mirror cell in the peripheral region
  \[
  \frac{\partial p U^\gamma}{\partial \psi} \geq 0 \text{ is satisfied because } \frac{\partial U}{\partial \psi} > 0.
  \]

The purpose of this research is to make clear the flute mode and associated transport in the GAMMA10 A-divertor with a mixed stability mechanism (min.B in the core and plasma compressibility in the periphery).

- In the steady state, a stable plasma pressure radial profile is plotted in the figure, where \( \frac{\partial p}{\partial \psi} < 0 \) in the core region and \( \frac{\partial p U^\gamma}{\partial \psi} \approx 0 \) in the periphery.
- However, there is a large ion loss near the separatrix because of large ion Larmor radius there, which breaks the relation \( \frac{\partial p U^\gamma}{\partial \psi} < 0 \) and makes the flute modes unstable.
Specific Volume \( U \) of the GAMMA10 A-divertor

In order to calculate the specific volume, the axial pressure profile is assumed to be

\[
\hat{p}(B) \equiv \hat{p}_\perp(B) + \hat{p}_\parallel(B) = \max \left\{ p_A \left( \frac{B_m^2 - B_c^2}{B_m^2 - B_c^2} \right), 1 \right\}, \quad U \equiv \int \frac{\hat{p}_\parallel(B) + \hat{p}_\perp(B)}{B} d\ell
\]

\( p_{\parallel,\perp}(\psi, B) = \hat{p}_{\parallel,\perp}(B) \nu(\psi) \), \( p_A \) is the pressure at the anchor midplane; \( B_c \) is the magnetic field at the midplane on axis in anchor cell and \( B_m = 1.7B_c \), that is anchor pressure becomes unity at \( B(z) = B_m \). The pressure \( p \) in the other region is assumed to be unity.

\( x = \sqrt{\psi/\psi_b} \), where \( \psi_b \) is the coordinate at the separatrix. In the figure, it is found

- In case of \( p_A < 6 \), there is no magnetic well. Thus \( pU^\gamma \approx \text{const.} \) must be satisfied in the whole radius.
- In case of \( p_A = 12 \), there is a magnetic well in \( x \lesssim 0.2 \). Thus \( \partial p/\partial \psi < 0 \) is permitted in \( x \lesssim 0.2 \).
- In case of \( p_A = 20 \), there is a magnetic well in \( x \lesssim 0.45 \). Thus \( \partial p/\partial \psi < 0 \) is permitted in \( x \lesssim 0.45 \).
Linear Growth Rate of \( m = 1 \) Flute Mode

A non-local calculation is required to determine the linear growth rate of a flute mode, because the flute instability is a global mode.

The non-local dispersion equation is derived from the basic equations derived on the assumption of axisymmetric magnetic field, which are used in the simulation and will be mentioned later in the presentation.

In order to solve the non-local dispersion equation, equilibrium quantities \( \hat{w}_0(x), \hat{\rho}(x), \hat{T}(x) \) are necessary. Here \( x = \sqrt{\psi/\psi_0} \), \( \hat{w}_0, \hat{\rho}, \hat{T} \) are normalized vorticity, mass density, and temperature.

Stable equilibrium : \( \hat{w}_0(x) = 1, \hat{\rho}(x) = 1, \hat{T}(x) = 1 \)

Note \( pU^y = \text{const} \) leads that \( \hat{\rho} = \rho U = \text{const} \) and \( \hat{T} = TU^{y-1}/M_i = \text{const} \) along \( x \).

\[ \hat{T}(x) = \exp(-2x^2) : \text{slim}, \quad \hat{T}(x) = \exp(-x^2/2) : \text{fat} \]
Linear analysis indicates that the linear growth rate of flute instability depends on the radial pressure profile as well as the radial profile of the specific volume of a magnetic field line. The actual radial pressure profile should be determined self-consistently by taking into account the classical diffusion and the diffusion caused by flute mode fluctuation.

Purpose

- Perform the simulation on a flute instability with the specific volumes of a magnetic field line in the figure. The code has the effects of classical viscosity and classical diffusion.
- Start at the divertor equilibrium i.e., \( pU' = \text{Const} \) radially.
- Compare the linear growing phase in the simulation with linear theory.
- Make clear whether the flute mode is unstable or not in the radial profile of \( U \).
- Examine the nonlinear saturation phase in the simulation and the radial transport.
Basic Equations used in the Simulation Code

Basic equations, which were derived by Dr. Pastukhov, assume an axisymmetric magnetic field.

Equation of motion

\[
\frac{\partial}{\partial t} \hat{w} + \{[\Phi, \hat{w}] - \{\hat{\rho}, \frac{\langle v^2 \rangle}{2}\}\} + \frac{1}{U} \frac{\partial U}{\partial \psi} \frac{\partial \hat{T}}{\partial \varphi} = \frac{3X_M}{20} \left( \frac{T_i + T_e}{2T_i} \right) \left\{ \frac{\partial}{\partial \psi} \left( \hat{\rho} \langle r^2 \rangle \frac{\partial}{\partial \psi} \left( \frac{1}{U^{2/3}} \sqrt{T_0} \frac{a^2}{\langle r^2 \rangle} \hat{w} \right) \right\} + \frac{a^2}{\langle r^2 \rangle} \frac{1}{U^{2/3}} \sqrt{T_0} \frac{\partial^2 \hat{w}}{\partial \varphi^2} + UQ_w^* \tag{1}
\]

Here \(\hat{w}\) represents the vortex defined by

\[
\hat{w} = \frac{\partial}{\partial \psi} \left( \hat{\rho} \langle r^2 \rangle \frac{\partial \Phi}{\partial \psi} \right) + \frac{\partial}{\partial \varphi} \left( \hat{\rho} \left( \frac{1}{r^2 B^2} + \lambda^2 B^2 \right) \frac{\partial \Phi}{\partial \varphi} \right) \tag{2}
\]

As a definition, \(\hat{A} = \langle A \rangle U\) is integral along a magnetic field line.

\[
\hat{A} \equiv \int A \frac{d\ell}{B}, \quad \langle A \rangle \equiv \frac{1}{U} \int A \frac{d\ell}{B}, \quad U \equiv \int \frac{d\ell}{B} \tag{3}
\]

\([\Phi, \hat{w}]\) is known as Poisson bracket defined by

\[
[\Phi, \hat{w}] \equiv \frac{\partial \Phi}{\partial \psi} \frac{\partial \hat{w}}{\partial \varphi} - \frac{\partial \Phi}{\partial \varphi} \frac{\partial \hat{w}}{\partial \psi} \tag{4}
\]

\([\Phi, \hat{w}]\) represents that the vortex flows through \(E \times B\)-drifts.
Electrostatic potential is determined by the following Poisson equation

\[
\frac{\partial}{\partial \psi} \left( \hat{\rho} \langle r^2 \rangle \frac{\partial}{\partial \psi} \Phi \right) + \frac{\partial}{\partial \varphi} \left( \hat{\rho} \left( \frac{1}{r^2 B^2} + \lambda^2 B^2 \right) \frac{\partial}{\partial \varphi} \Phi \right) = \hat{\omega}
\]

Equation of continuity

\[
\frac{\partial}{\partial t} \hat{\rho} + [\Phi, \hat{\rho}] = 4\pi \frac{\partial}{\partial \psi} \left( \hat{\rho} \langle r^2 D \rangle \frac{\partial p_0(\psi)}{\partial \psi} \right) + Q^s \rho U
\]  

(5)

Equation of temperature transport (line temperature \([\hat{T} \equiv pU^\gamma/\hat{\rho} = (T_e + T_i) U^{\gamma-1}/M_i]\))

\[
\frac{\partial}{\partial t} \hat{\rho} + [\Phi, \hat{T}] = - (\gamma - 1) \frac{U^\gamma}{\hat{\rho}} \left\{ \frac{1}{U} \frac{\partial}{\partial \psi} \left( U \langle q \cdot \nabla \psi \rangle \right) + \frac{\partial}{\partial \varphi} \langle q \cdot \nabla \varphi \rangle \right\} \\
+ 4\pi \left[ \frac{\partial}{\partial \psi} \langle r^2 D \rangle \frac{\partial p_0}{\partial \psi} \right] + 4\pi (\gamma - 1) \hat{T} \frac{\partial}{\partial \psi} \left( \langle r^2 D \rangle \frac{\partial p_0}{\partial \psi} \right) + Q^T \frac{U^\gamma}{\hat{\rho}}
\]  

(6)

Those basic equations contain the interchange modes (similar to the Rayleigh-Taylor instabilities) and the modes associated with the presence of nonuniform plasma flows (similar to the Kelvin-Helmholtz instabilities) as well as the electrostatically incompressible stable plasma flows. So this close set of equations describe the nonlinear low-frequency MHD plasma convection and resulting transport processes in weakly dissipative plasmas in axisymmetric shearless systems.
Initial Condition and Boundary Condition of the Simulation

We solve the two-dimensional variable \( \hat{w}(x, \varphi) \), \( \Phi(x, \varphi) \), \( \hat{\rho}(x, \varphi) \), and \( \hat{T}(x, \varphi) \). \( x = \sqrt{\psi/\psi_b} \), \( \psi_b \) is the coordinate at the separatrix. \( \varphi \) is the angle coordinate. As already mentioned, quantities with \( \hat{\cdot} \) denote the quantities integrated along a magnetic field line. Here the GAMMA10 A-divertor vacuum magnetic field is used in the simulation.

Boundary condition

\[
\frac{\partial \hat{\rho}(x = 1, \varphi)}{\partial x} = 0, \quad \frac{\partial \hat{T}(x = 1, \varphi)}{\partial x} = 0, \quad \Phi(x = 1, \varphi) = 0.
\]

\( \hat{w} \) is programmed so as to conserve \( \int_0^1 x dx \int_0^{2\pi} d\varphi \hat{w}(x, \varphi) \) in time.

All variables used in the simulation are normalized.

Initial condition

\[
\hat{\rho}(x, \varphi) \equiv \rho(x, \varphi) U(x) = 1, \quad \hat{T}(x, \varphi) \equiv T(x, \varphi) U(x)^{2/3}/M_i = 1
\]

And \( \hat{w}(x, \varphi) = +1 \) is assumed initially, which gives the initial \( \Phi \).

First of all, the unstable case of \( p_A = 1 \) in the GAMMA10 A-divertor is shown.
The time evolution of potential $\Phi$ is plotted in right figure. Fourier amplitudes of the potential $\Phi(m)(x)\exp\{im\varphi\}$ observed at $x = 1/2$ are plotted. The flute mode is unstable in this specific volume $U(x)$ case and grows linearly about $\tau \approx 80$. The solid line in the figure is the growth rate obtained by non-local linear analysis by using $\rho_0$ and $T_0$ observed in the simulation at $\tau = 80$. Agreement of simulation with linear analysis is good. At $\tau = 0$, the simulation start at the divertor equilibrium state, that is $\hat{\rho} = \text{const}$ and $\hat{T} = \text{const}$. A large classical transport around $x = 1$, where is the separatrix, makes the pressure profile unstable to the flute modes like right figures. Figure (c) is the eigen-function of $\Phi$ of $m = 1$ observed in the simulation. The eigen-function of $\Phi$ of $m = 1$ obtained by the non-local linear analysis is plotted in Fig.(d). Agreement of Fig.(c) and (d) is very good.
Linear Growing Phase of $p_A = 1$

These figures are equi-contour plots of $\tilde{\Phi}(x, \varphi)$, $\tilde{w}(x, \varphi)$, $\tilde{\rho}(x, \varphi)$, and $\tilde{T}(x, \varphi)$ observed at $\tau = 80$ (linear phase). It is seen that the dominant mode number is $m = 1$ in all quantities and those are localized around $x \approx 1/2$. Here all quantities are subtracted $m = 0$ component.

The divertor mirror has a magnetic null surrounding a magnetic axis. The magnetic null has an effect that unmagnetized electrons can move freely in the azimuthal direction. Thus the boundary condition $\hat{\Phi}(x = 1, \varphi) = 0$ is adopted in the simulation. The classical transports of density, temperature, and vorticity are also large in the azimuthal direction around magnetic null because of large Larmor radius. All of the density, temperature, and vorticity have the equi-contour around $x = 1$ (separatrix).

Plasma rotates counter-clockwise with frequency $\omega \approx 2.1$, which agrees well with that determined by the non-local linear analysis. The power spectrum of $\tilde{\Phi}$ is plotted. It is seen that the higher modes are very small in all $x$. 

\[ \Phi \sim w \sim T \sim \tau = 80 \quad P_A = 1 \]

\[ \max = 3.0 \times 10^{-4} \quad \max = 7.2 \times 10^{-4} \quad \max = 3.6 \times 10^{-6} \]

at $x = 1/5$ at $x = 2/5$ at $x = 4/5$
Non-linear Saturation Phase of $p_A = 1$

The flute instability saturates at $\tau \approx 100$. A non-linear saturation state is realized after $\tau \approx 100$. The largest mode is $m = 1$. All modes are repeating the grow up and down of Fourier amplitude in time. Those periods are several 10 normalized time, which are comparable to the growing time in linear phase.

The eigen-functions of $m = 1$ Fourier amplitude are localized around $x \approx 2/5$ in the linear phase. From $\tau = 100 \rightarrow 130$ the $m = 1$ Fourier amplitude decreases in time. The eigen-functions of $m = 1$ Fourier amplitude in these time duration is plotted in right figures. At $\tau = 100$ the eigen-function is localized around $x \approx 2/5$, which is just like that in the linear growing phase.

It is seen that a peak point of the eigen-function moves outward when the Fourier amplitude decreases in time from $\tau = 110 \rightarrow 130$. At the time when the Fourier amplitude is local maximum, each eigen-function at $\tau = 210$, $260$, $300$, and $330$ has its peak around $x \approx 2/5$. Thus in the non-linear phase flute instabilities repeat the same process of grow up and down.
Non-linear Saturation Phase of $p_A = 1$

These figures are equi-contour plots of $\tilde{\Phi}(x, \varphi)$, $\tilde{w}(x, \varphi)$, $\tilde{\rho}(x, \varphi)$, and $\tilde{T}(x, \varphi)$ observed at $\tau = 100$ (First saturation phase). That is, flute instabilities enter the non-linear saturation phase at $\tau = 100$. The contour surfaces are the vortex $\tilde{w}$, $\tilde{\Phi}$, $\tilde{\rho}$, and $\tilde{T}$, respectively. Here all quantities are subtracted $m = 0$ mode. These countor plots are almost the same as those at the linear growing phase $\tau = 80$.

The remarkable character is that the eigen-function has a peak around $x \simeq 1/2$.

The power spectrum of $\tilde{\Phi}$ is plotted. It is seen that the higher modes at $x = 2/5$ are smaller that those in $x = 4/5$. However, the power spectrum becomes broader at $x = 4/5$ than $x = 2/5$ at $\tau = 130$. 
Non-linear Saturation Phase of $p_A = 1$

There contour plots are more clear. The perturbations localized around $x \approx 1/2$ at $\tau \approx 100$ shift to the peripheral region at $\tau \approx 130$.

These figures show that the flute instability is saturated around $x \lesssim 3/5$ after $\tau = 100$, while the large flute mode fluctuations still exist around $x \approx 4/5$ at $\tau = 130$ in the simulation.
Flute Instability and Associated Transport

The time variations of two spacial points are plotted in the figures. Two special points are chosen at \((x = 0)\) and \((x = 1/3, \varphi = 0)\).

The time variations of all quantities are lost by the classical radial transport in a linear phase \(\tau < 100\). A remarkable feature is that a very large transport occurs at \(\tau \sim 100\), when the Fourier amplitude is maximum. The time when the large transport occurs coincides with the time when \(m = 1\) Fourier amplitude of \(\Phi\) has maximum in time.
Flute Instability and Associated Transport

The enhanced radial transport due to the flute instabilities is seen more clearly in the figures. The flute modes grow up to its maximum of $m = 1$ Fourier amplitude at $\tau \approx 100$. The radial transport associated with the flute instability is plotted in the figures as a function of radius $x$.

The anomalous transport results from the terms which is $\propto \int_0^{2\pi} (\hat{\rho} \partial \Phi / \partial \varphi) \, d\varphi$ for density and $\propto \int_0^{2\pi} (\hat{T} \partial \Phi / \partial \varphi) \, d\varphi$ for temperature.

In the neighborhood of $x = 1$, classical transport is large because of large Larmor radius around magnetic null.

It is found that anomalous transport is much larger than the classical transport in the core region at $\tau = 100$. But it is lower than the classical one at $\tau = 130$. That is, flute instability enhances the radial transport when its Fourier amplitude becomes large.
The reason that the flute instability saturates at $\tau \approx 100$ can be seen in the right figures. At $\tau = 90$ in the linear growing phase of flute modes, the relation $\partial p U^\gamma / \partial \psi < 0$ satisfies the condition that the flute modes are unstable. However, from $\tau = 100$ to $120$ the slope of $\hat{T}$ becomes flat as a result of large radial transport appearing at this time. As already mentioned in the presentation, the eigen-mode of $m = 1$ flute instability has a peak point which moves radially in time $\tau = 100$ to $120$. As the temperature becomes flat in the core region, the slope of the temperature becomes steep in the peripheral region, which enhances the classical transport there. So again flute modes become unstable, which is repeated periodically.
In the case without high pressure in an anchor mirror, that is, without min.B mirror effects to stability, the flute instability in the divertor mirror was studied to the effect on the radial transport. Now I will present the effect of anchor mirror cell to the flute instability in the divertor mirror. In the case that the maximum pressure at anchor midplane is twenty times larger than that in the central, the specific volume has the radial profile in the figure. That is, magnetic well is created in the region \( x \lesssim \frac{1}{2} \).

The simulation result is plotted in the right figures. Fourier mode of potential grows in time. However, the growth rate of the \( m = 1 \) mode do not agree with the linear theory. The eigen-function of \( m = 1 \) mode of \( \Phi \) is quite different from the linear theory. Therefore, we conclude that these modes are not the unstable modes, but these are thermal fluctuations of initial perturbations, because the modes decrease after \( \tau \approx 280 \).
Effects of Min.B Anchor Cell to the Flute Instabilities

The radial profiles of density and temperature are plotted at time τ = 320.
Radial profiles do not have a flat region, that is

\[ pU^γ \neq \text{const} \quad \text{in whole region of } x \]

This radial profile can make flute modes unstable if there is no min.B.
A remarkable feature is that \( \hat{\rho}_0 \hat{T}_0 \) has a Gaussian type radial profile, which is clearly stabilized by min.B of

\[ \frac{\partial U}{\partial x} < 0, \quad \text{in the region } x \lesssim 0.45 \]

Right figures are the time variation of \( \Phi, \hat{\rho}, \hat{T} \) observed at \((x, \varphi) = (1/3, 0)\) in the simulation. The dotted lines is the case without flute perturbations, that is, only classical transport is included in the simulation.
It is found that in case of \( p_A = 20 \) radial transport is almost classical diffusion, because both cases have the same time variations of \( \hat{T} \) and \( \hat{\rho} \).
Effects of a Magnetic Well on the Flute Instabilities

Pressure $p_A$ in the anchor mirror cell changes the radial profile of specific volume $U$ as shown in the figure. If $p_A \gtrsim 12$, there is a region $\frac{\partial U}{\partial x} < 0$ around $x \approx 0$, that is, there is a magnetic well around axis. The maximum amplitude of $m = 1$ Fourier component $\tilde{\Phi}$ observed at $x = 2/5$ in a simulation run is plotted as a function of $p_A$ of several simulation runs. The magnetic well has an effect of making the maximum amplitude of flute instability lower. And it is found in the simulations that if $p_A \gtrsim 20$, flute modes are stabilized by a magnetic well.

The figure, at the bottom of right side figures, plots the linear growth rate of flute instability observed in the simulation as a function of $p_A$ by triangles. The linear growth rates of flute instabilities determined by non-local linear theory are also plotted in the figure by circles. Agreement between simulation and theory is very good for various parameters of $p_A$. 

\begin{align*}
\quad &\text{Pressure } p_A \text{ in the anchor mirror cell changes the radial profile of specific volume } U \text{ as shown in the figure. If } p_A \gtrsim 12, \text{ there is a region } \frac{\partial U}{\partial x} < 0 \text{ around } x \approx 0, \text{ that is, there is a magnetic well around axis.} \\
\quad &\text{The maximum amplitude of } m = 1 \text{ Fourier component } \tilde{\Phi} \text{ observed at } x = 2/5 \text{ in a simulation run is plotted as a function of } p_A \text{ of several simulation runs. The magnetic well has an effect of making the maximum amplitude of flute instability lower. And it is found in the simulations that if } p_A \gtrsim 20, \text{ flute modes are stabilized by a magnetic well.} \\
\quad &\text{The figure, at the bottom of right side figures, plots the linear growth rate of flute instability observed in the simulation as a function of } p_A \text{ by triangles. The linear growth rates of flute instabilities determined by non-local linear theory are also plotted in the figure by circles. Agreement between simulation and theory is very good for various parameters of } p_A. 
\end{align*}
Summary

• The GAMMA10 A-divertor has a minimum-B mirror cell and an axisymmetric divertor mirror cell. Although both cells have a magnetic geometry stable to flute modes, the stability mechanism is different with each other.
  - Minimum-B mirror is to utilize a good magnetic field line curvature.
  - Divertor mirror is to utilize a plasma compressibility.
  Divertor has further an effect of making the separatrix magnetic flux tube equi-potential, where the potential fluctuations are stabilized.
• The purpose of this presentation is to make clear the flute instability and associated radial transport in the mixed stabilizing mirror cells of minimum-B and divertor.
• In order to apply the basic equations (derived in a axisymmetric system) to apply the non-axisymmetric GAMMA10 A-divertor, the definition $U$ is changed as

$$U = \int \frac{d\ell}{B} \quad \Rightarrow \quad U = \int \frac{(\hat{p}_\parallel + \hat{p}_\perp)}{B} d\ell$$
Summary

- Case without high pressure in minimum-B mirror. Flute modes are stabilized by only divertor mirror cell.
  - The flute modes are found to be always unstable.
  - Although there is a radial pressure profile stable to flute modes in the divertor mirror, a large radial transport around x-point breaks the stable pressure radial profile and makes flute modes unstable.
  - The flute instability fortunately (unfortunately) is not so strong to destroy the plasma immediately. Instead the radial transport is enhanced by the intermittently growing flute mode fluctuations.
  - Rapid evacuation of plasma to the dipole region is required in a divertor experiment of GAMMA10 A-divertor. Therefore, further consideration is required in order to understand whether the radial transport caused by the flute modes in a non-linear saturation phase can meet the requirement or not.
- Case with high pressure in minimum-B mirror. In this case flute modes are stabilized by both divertor and minimum-B mirror cells.
  - Minimum-B mirror is found to have a tendency of stabilizing the flute modes even with the divertor mirror cell.
  - However even if there is a magnetic well around axis, which is stable to flute modes in a long thin approximation, flute modes are unstable as long as the well is not so deep. Thus higher pressure in anchor mirror than the present GAMMA10 is required to stabilize the flute modes completely.
  - If flute modes are stabilized, there can be realized that a pressure has a Gaussian radial profile within the magnetic well and has a divertor stable radial profile outside the magnetic well.
Ion possible region of motion in a dipole region

Ion motion in an axi-symmetric mirror is described in terms of pseudo-potential as

$$M_i \ddot{r} = -\frac{\partial \Phi}{\partial r}, \quad M_i \ddot{z} = -\frac{\partial \Phi}{\partial z}$$

$M_i$ is ion mass, and $\Phi$ is pseudo-potential

$$\Phi \equiv \frac{q^2}{2M_ic^2} \left( \frac{\psi - \psi_0}{r} \right)^2$$

$\psi, \theta$ are defined as $B = \nabla \psi \times \nabla \theta$.

Figure plots the equi-potential contours of $\Phi$, where 1keV ions and 9keV ions are confined within the potential. Ions with its kinetic energy lower than 9keV are confined inside the dipole region.