Equilibrium of a High-β Plasma with Sloshing Ions Above the Mirror Instability Threshold

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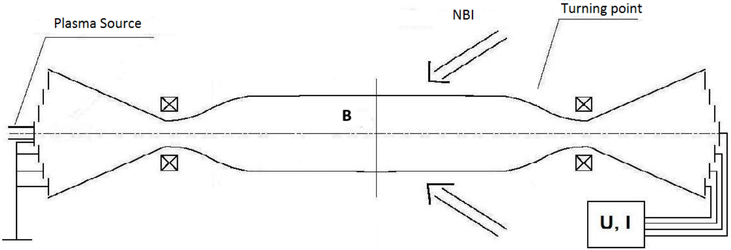
Slope NBI

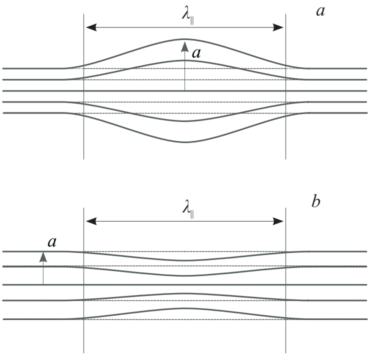


Slope Neutral Beam Injection produces a population of fast sloshing ions. Transversal plasma pressure exhibits narrow picks near the turning points of the sloshing ions.

Mirror Instability ($p_\perp \gg p_\parallel$)

An occasional rarefaction of magnetic field lines forms a mirror cell (Fig. a). The cell retracts an extra portion of the plasma. As the plasma pressure $p_\parallel$ increases to balance the depression of the magnetic pressure $p_M = B^2/8\pi$, the total transversal pressure $p_\perp + p_M$ increases, making the rarefaction unstable. The magnetic field depression deepens, if $\delta p_\perp + \delta p_M > 0$ for $\delta B < 0$.

Similarly, a local compression of B forms an unstable mirror throat (Fig. b).



Mirror Instability Threshold

A. Vedenov, L. Rudakov and R. Sagdeev (end of 1950s).

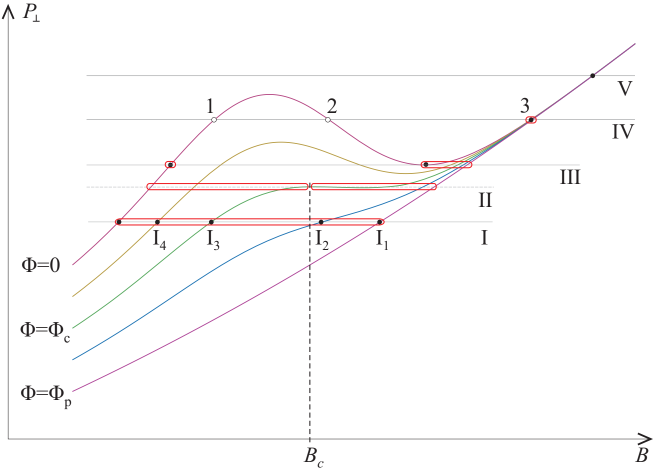
A stability criterion for the mirror mode in the present form was formulated by W. Tompson (1964):

$$\frac{\partial}{\partial B}\underbrace{\left(p_\perp + \frac{B^2}{8\pi}\right)}_{\equiv P_\perp} > 0.$$

H. Grad (1967) argued that the same condition is necessary for correct formulation of the boundary value problem for plasma equilibrium in an open trap.

I. Lansky (1993) believed that this condition also restricts the applicability of the paraxial approximation in an open trap by the case, where the plasma pressure is sufficiently small.

K. Lotov gave an example (1996) of paraxial equilibrium, where a magnetic hole with exactly zero magnetic field was formed inside the plasma.



Plot of $P_\perp(B)$ vs. B is not monotonic if the mirror instability threshold is exceeded.

Basic Equations

Within paraxial approximation transversal equilibrium is govern by the reduced equation,

$$p_\perp(\Phi, B) + \frac{B^2}{8\pi} = \frac{H^2(z)}{8\pi}, \tag{1}$$

instead of

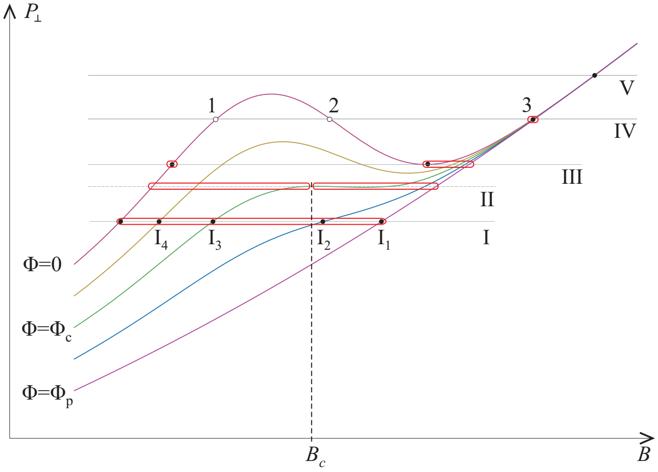
$$\frac{\partial}{\partial n}\left(B^2 + 8\pi p_\perp\right) = \kappa\left(2B^2 + 8\pi p_\perp - 8\pi p_\parallel\right).$$

In the same approximation,

$$\frac{\partial r^2}{\partial \Phi} = \frac{1}{\pi\, B(\Phi, H)}. \tag{2}$$

Outside the plasma $B(\Phi, H) = H$. One needs to find a solution on the interval $0 \leqslant \Phi \leqslant \Phi_p$, where $p_\perp > 0$.

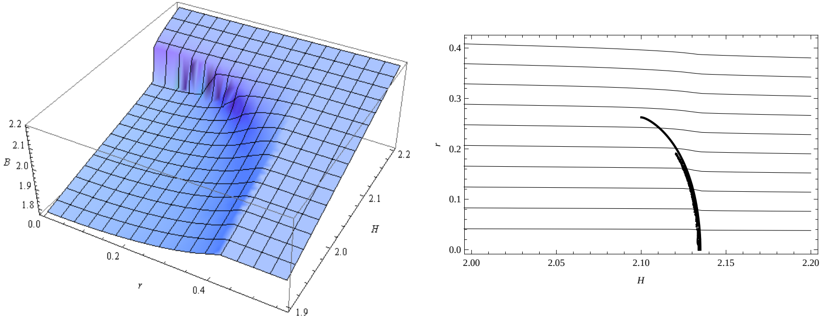
Graphical solution



$$p_\perp(\Phi, B) + \frac{B^2}{8\pi} = \frac{H^2(z)}{8\pi}.$$

Magnetic Hole

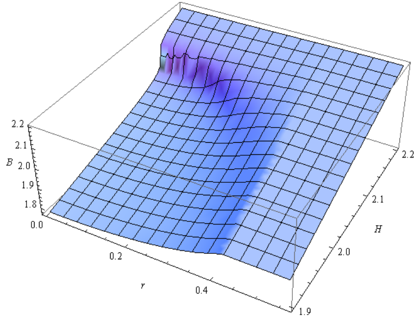
Deep Hole

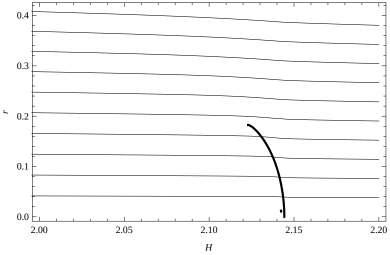


$\Delta b = 0.10,\ \beta_{\perp 0} = 0.05,\ b_* = 2,\ \Phi_p = 1,\ B_0 = 1$

Magnetic Hole

Shallow Hole

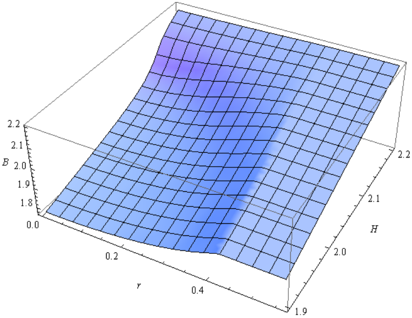


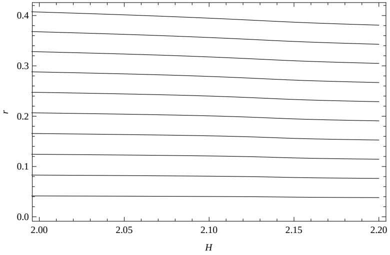


$$\Delta b = 0.12,\ \beta_{\perp 0} = 0.05,\ b_* = 2,\ \Phi_p = 1,\ B_0 = 1$$

Magnetic Hole

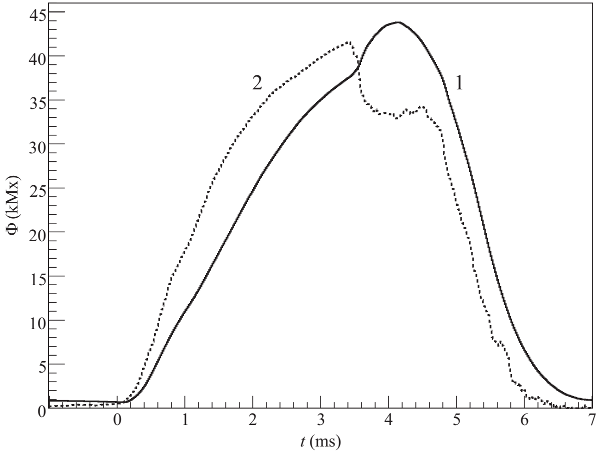
No Hole





$$\Delta b = 0.15,\ \beta_{\perp 0} = 0.05,\ b_* = 2,\ \Phi_p = 1,\ B_0 = 1$$

Diamagnetic Signal



1 — GDT midplane, 2 — turning point of sloshing ions
[Kotelnikov, Bagryansky, Prikhodko, Phys. Rev. E **81**, 067402 (2010)]

Shallow Magnetic Hole

$P_\perp(B,\Phi)$ is expanded into the Taylor series up to the cubic terms in $(B - B_c)$ around the critical point B_c, Φ_c, where

$$\frac{\partial P_\perp}{\partial B} = 0, \qquad \frac{\partial^2 P_\perp}{\partial B^2} = 0.$$

- The problem is solved in an analytical form.
- Its solution uncovers the limits of applicability of the paraxial approximation for the plasma equilibriums with magnetic hole.

Conclusions

- The non-uniformity of the magnetic field stabilizes the mirror instability (in a certain sense).
- Above the mirror instability threshold, there are two equilibrium states with magnetic holes of different depth. A bifurcation between these two equilibriums is accompanied by ejection of some portion of the plasma from the magnetic hole to the central part of the trap.
- Present model does not account for the effect of Finite Larmor Radius.
- Numerical simulation is required to account for non-paraxial effects.